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BEHAVIOR OF POSITIVE IONS IN HYDROGEN.

BY

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ABSTRACT.

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It has been shown that electrons with 11.5 volts energy can dissociate a hydrogen molecule into its two constituent atoms, but up until recently no experiments have been performed to see whether fast positive ions are able to dissociate hydrogen.

In the present experiments Li or Cs positive ions of various velocities are produced in a tube containing hydrogen molecules. The tube is immersed in liquid air, and the rate of decrease of pressure of hydrogen is measured as a function of the velocity of the positive ions. It is found that with no positive ions entering the tube there is a certain decrease in the pressure of the hydrogen due to its thermal dissociation on the hot filament and its subsequent condensation on the cold walls of the tube. With ions of energies from 15 to 320 volts flowing in the tube the rate of decrease of pressure is greater, showing a formation of some condensible product due to the action of the ions.

The rate of pressure decrease with the voltage applied has been found to be proportional to the ion current flowing. The rate per unit current is proportional to the pressure; and the rate per unit current per unit pressure is practically independent of the voltage for Li and Cs ions of energies from 15 to 320 volts.

The effect has been shown not to be due to secondary electrons.

The process may be due to the formation of alkali hydrides in the gas phase. *The number of hydrogen molecules disappearing per positive ion entering the tube varies from 0.01 to 0.5. No critical potentials have been found in this experiment, in disagreement with recent experiments of Leipunsky and Schechter.*

INTRODUCTION.

It is well known that molecular hydrogen can be activated in a number of ways and made to react with various sub-

stances. Since most problems of molecular physics are of a more complicated nature than the pure atomic phenomena their solution is more difficult and sometimes does not lead to unique results. The problem of activation of hydrogen is even more difficult to solve as its activation cannot be directly detected spectroscopically and is usually detected by chemical means, or as a result of measurements on rates of chemical reactions.

There are, however, some outstanding facts known about the activation of hydrogen. Leaving out the field of surface activation (catalysis) there are three known methods of activation of hydrogen.

1. *Thermal Activation:* The hydrogen molecule picks up energy of vibration due to thermal energy. If the temperature is high enough the molecule can pick up sufficient vibrational energy to dissociate into two hydrogen atoms. This phenomenon has been investigated by Languir[1] who has been able to dissociate hydrogen on a hot tungsten wire and to measure its heat of dissociation. A somewhat more exact value of the heat of dissociation has been determined spectroscopically by Witmer[2] who has found the value 4.3 volts.

2. *Activation by Collision of the First Kind with Electrons:* It has been shown by Hughes[3] and by Glockler, Baxter and Dalton[4] that hydrogen may be dissociated by electrons having about 11.4 volts energy. The latter have made a search for dissociation due to electrons having energies from 4 to 10 volts but could find no evidence that any dissociation occurs below 11.4 volts. Recently Dorsch and Kallmann[5] have preformed an experiment in which they believe that they have obtained evidence for dissociation by electron impact at energies as low as 7 volts. The probability of such an occurrence is, according to these investigators, extremely low.

Condon and Smyth[6] have interpreted the experiments of Hughes, and Glockler, Baxter and Dalton by considering the potential energy curves of the hydrogen molecule as drawn

[1] I. Langmuir, *Jour. Amer. Chem. Soc.*, **34**, 1310 (1912), **37**, 417 (1915), **38**, 221 (1916).

[2] E. E. Witmer, *Proc. Nat. Acad. Sci.*, **12**, 238 (1926).

[3] A. L. Hughes, *Phil. Mag.*, **41**, 778 (1921).

[4] Glockler, Baxter, and Dalton, *Journ. Amer. Chem. Soc.*, **49**, 58 (1927).

[5] Dorsch and Kallmann, *Zs. f. Phys.*, **53**, 80 (1929).

[6] Condon and Smyth, *Proc. Nat. Acad. Sci.*, **14**, 871 (1928).

by Winans and Stueckelberg.[7] For the above experiments three energy states are to be considered; the lowest state (1^1S), having a well defined potential energy minimum; the 1^3S state having no potential minimum; and a 2^3S state having a minimum. The 2^3S and 1^3S states have practically the same potential energy when the nuclear separation is that of the minimum of the 1^1S curve. The energy difference between the lowest state and the two higher states at this point is about 11 volts. The only transition the molecule can make on electron impact below 11 volts is from the 1^1S to the 1^3S state. Applying the Franck-Condon Principle (certainly applicable for light quants and electrons), which states that the most probable transition in the molecule is that which makes the change in nuclear separation a minimum, it is easy to show that the most probable transition is from the 1^1S to the 1^3S states at about 11 volts. Since the 1^3S state has no potential minimum, the molecule will fly apart into two hydrogen atoms. At lower voltages, transitions may occur in which the nuclear separation is altered and are therefore less probable, in agreement with the above mentioned experiment.

3. *Activation by Collision of the Second Kind:* A third type of activation is that by collision of the second kind between an excited atom and a hydrogen molecule. It is well established that an excited mercury atom with 4.9 volts energy may activate a hydrogen molecule and make it react with various substances. This has been shown first by Cario and Frank[8] and has been verified by many other investigators. The mechanism of this activation is still somewhat obscure. Franck and others have assumed the process involved the formation of two atoms of hydrogen in an elementary act. Up to the present no very definite proof has been given of this. since most of the evidence going to show that atomic hydrogen is formed on data obtained from rates of reactions and involves many doubtful assumptions. Compton and Turner[9] assume that the result of the collision is an H atom and a molecule of HgH, and they have evidence that this occurs in the low volt-

[7] Winans and Stueckelberg, ibid., **14**, 867 (1928).

[8] G. Cario and J. Franck, *Zs. d. Phys.*, **11**, 161 (1922).

[9] K. T. Compton, and L. A. Turner, *Phil. Mag.*, **48**, 360 (1924).

age arc. The author [10] has some evidence to the fact that molecular hydrogen might take up energy of vibration from the excited mercury atom.

4. *Activation by Positive Ion Impact:* Up until very recently no work has been done on the activation of hydrogen by fast positive ions. In order to throw further light on the mechanism of the activation of hydrogen and as to what effect the mass of the striking ion has on the process, the writer de-

FIG. 1.

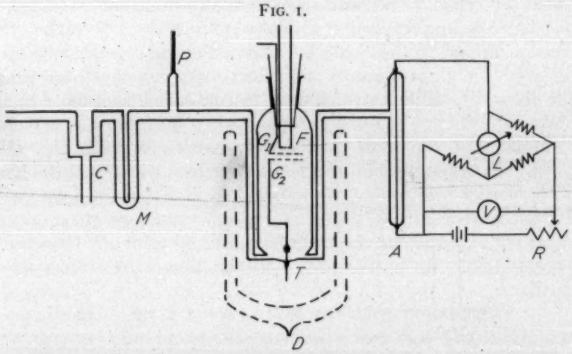


Diagram of apparatus.

signed the present experiment to see whether molecular hydrogen could be dissociated by positive ion impact and, if so, to determine at what voltages the dissociation occurred. While the experiments were in progress a similar experiment was performed by Leipunsky and Schechter [11] which showed that hydrogen could be activated by fast positive ions and that the voltage at which activation occurred depended on the mass of the positive ion. They concluded that the most probable mechanism was the excitation of the molecule to the resonance level (11.6 volts) and its subsequent decomposition.

Since the present experiments of the writer are in disagreement with those of Leipunsky and Schechter and since in one fespect their experiments lead to some difficulties, it was thought worth while to report the present experiments.

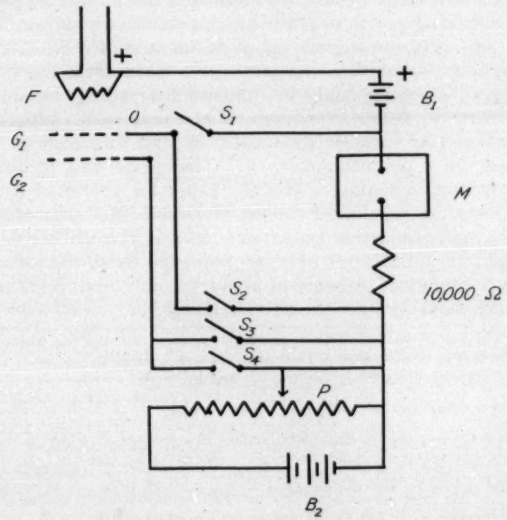
[10] A. C. G. Mitchell, *Proc. Nat. Acad. Sci.*, **11**, 458 (1925).

[11] A. Leipunsky and A. Schechter, *Zs. f. Phys.*, **59**, 857 (1930).

EXPERIMENTAL ARRANGEMENT AND METHOD OF PROCEDURE.

The experimental arrangement (Fig. 1) was similar in principle to that of Hughes, and Leipunsky and Schechter. The tube T, 5 cm. in diameter and 10 cm. long, was made of Pyrex glass and contained a filament, F, a grid G_1 of nickel and in some experiments a second grid G_2. The filament and grid G_1 were mounted on the same stem and were sealed into the tube by means of a ground glass joint. At the bottom of the tube there was a small rod which could be put at the same potential as G_1 leaving the space between them field free. In experiments where two grids were used the second grid G_2, was mounted on the tungsten rod at the bottom of the tube without any difficulty. The electrical connections used in experiments with two grids will be described later when those experiments are discussed. In most of the experiments, however, only the one grid was used. The distance from the filament to G_1 was about 2 mm. and from G_1 to the tungsten rod was 6 cm.

FIG. 2.



Wiring diagram for experiments with two grids.

The source of positive ions was a heated filament coated with the mineral spodumene (Li Al (SiO_3), to give Li ions, or Pollucite (H_2O, Cs_2O. $2Al_2O_3.9SiO_2$) to give Caesium ions.[12] The minerals were powdered and put on a small strip of platinum mesh with Zapon varnish. They were not heated in the air but were outgassed by heating in a vacuum at rather high temperature and while quite large currents (0.1 milliampere) were being drawn from them. During this process the mineral fused to the platinum strip and had no tendency to drop off. This gave a good steady source of ions.

The tube T had two glass tubes leading out of it from the bottom, one of which went to a Pirani gauge, A, and the other to a palladium tube, P, for letting in pure hydrogen, a small liquid air trap, M, and to the mercury cut off, C. The other side of the cut off lead to the high vacuum system and McLeod gauge. The tube T, could be immersed in liquid air up to the neck with the help of the Dewar flask D. The total volume of the system up to the cut off was about 300 cc. The Pirani gauge was used according to a method described by Campbell [13] for measuring pressures, and the connections are shown in the diagram. By putting the gauge in one arm of a Wheatstone bridge, the other three resistances of which were fixed, any change of pressure could be followed by reading the voltage necessary to obtain a balance. The voltage was changed by means of the variable resistance, R, and was read to four figures on a potentiometer, V. The gauge was calibrated with hydrogen against a McLeod gauge for a series of hydrogen pressures. In order to measure small differences of pressure a more sensitive galvanometer was thrown across the bridge after a balance had been obtained. With this arrangement a change of pressure of about 5×10^{-5} mm. correspond to 1 cm. deflection on the galvanometer.[14]

[12] The use of these minerals as sources of positive ions was first pointed out to the writer by Professor A. J. Dempster, of the University of Chicago.

[13] Campbell, *Proc. Phys. Soc. London*, 33, 287 (1921).

[14] According to this method of using the gauge $\frac{V^2 - V_0^2}{V_0^2} = \alpha p$ where V and V_0 are the voltage reading corresponding to a pressure p and to the highest vacuum obtained in the apparatus respectively, and α is a constant. By differentiation it can be shown that $\Delta p = \frac{2}{\alpha} \frac{V \Delta V}{V_0^2}$. Thus it is seen that Δp is a known function α, V, and V_0, and can be calculated for any V.

The experimental procedure was as follows: After pumping out the system to a pressure of 10^{-6} mm., the filament was heated and a current of 0.1 milliamperes drawn to G_1 using a potential of from 160 to 320 volts to accelerate the positive ions, for about 30 minutes to outgas the filament and the grid. The tube was then baked out at 400° C. for two hours and was then ready for a series of runs. Liquid air was always kept on the trap, M, so that no mercury vapor could get into the Pirani gauge, which was also heated from time to time and the filament flashed to keep it in condition. After the apparatus was baked out, liquid air was placed about T, hydrogen let into the apparatus through the palladium tube, and the cut off closed. The filament was then heated, and it was found that the pressure of the hydrogen decreased with the time, presumably due to the thermal formation of atomic hydrogen which "cleaned up" on the cold walls. This constituted a zero run and was taken alternately with all runs in which voltage was applied. After a zero run was taken various voltages were applied to accelerate the ions from F to G_1, and the pressure was read every minute to see whether a change in the rate of pressure decrease occurred when the voltage was applied, due to the formation of atomic hydrogen or some other condensible product. The current flowing was kept as constant as possible as was also the liquid air level. The initial pressure of hydrogen varied from 0.01 to 0.06 mm.

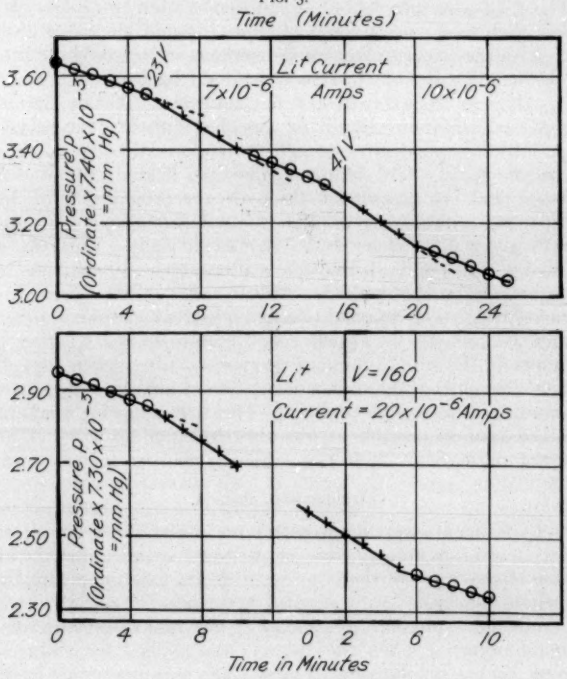
EXPERIMENTAL RESULTS.

Experiments were made with both Li ions and Cs ions but since, curiously enough, they showed no marked difference in their effects most of the later experiments were done with Cs ions.

It was found that an increase in the rate of pressure decrease occurred when the voltage was applied for voltages from 160 to 15 volts with Li ions. The currents varied from 30 to 6 microamperes. Below 15 volts no change in the rate could be detected, but at this low voltage the current was so low that one could not expect much of a change. Typical runs in which the pressure of hydrogen is plotted against the time are shown in Fig. 3. It will be noted that sharp breaks occur in the curve whenever the voltage is turned off or on.

The voltage was usually on for five minutes and off for five minutes alternately. Similar effects were found using Cs ions having velocities corresponding to voltages from 320 to 20, when the current flowing varied from 50 to 6 microamperes.

FIG. 3.



Experiments were now performed to see how that part of the rate depending on the positive ions varied with the current when the voltage and pressure were kept constant. The rate in millimeters per minute can be obtained from curves similar

to those of Fig. 3 by taking the difference of the slopes of the curves when the voltage is on and when it is off, or it can be calculated from the average change in voltage of the potentiometer reading by using the formula given in note 12. Table I shows that in two experiments where the current

TABLE I.

Rate (mm./min./amp.).	Current (amps. $\times 10^{-8}$).	Voltage.	Pressure (mm.).
4.6	67.0	160	0.044
5.0	16.7	160	0.044

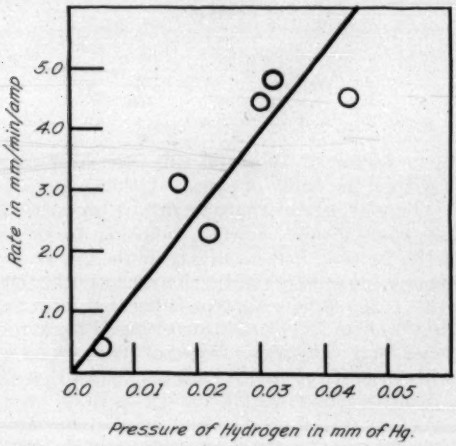
varied by a factor of four the rate per unit current was constant within the limit of error. Other experiments bear this out. Thus it is seen that the rate is proportional to the number of positive ions flowing, pointing to the fact that whatever the process, it is occurring in the gas phase.

At this point it was thought well to see whether the process could be due to secondary electrons liberated from the grid by positive ion bombardment, and accelerated back toward the filament with large velocities. Most of the work on secondary electrons liberated from metal surfaces shows that under the action of positive ions of as low energy as those used in these experiments very few, if any, secondary electrons are emitted. Since, however, most of the work on secondary emission has been done on specially treated surfaces several experiments were designed to see if secondaries could be causing the effects noticed when the voltage was applied in the present tube.

In the first set of experiments a copper disk, the same size and shape as the grid, was fastened to the filament leads 3 mm. behind the filament and was kept at the same potential as the filament. Small holes had to be cut in the copper disk in order not to short circuit the filament current, but the area of the disk was large compared to the holes so that any electrons coming from the grid 5 mm. away, would have a tendency to be stopped by the plate if their mean free path were greater than 5 mm., in which case one should expect a decrease in the rate of pressure decrease when the voltage was applied compared to when the disk was not present. If the mean free path were long compared to 5 mm. one might expect that there

would be no change, or only a slight one, when the voltage was applied if the effect is due to secondary electrons. Now the mean free path of an electron in hydrogen at 0.05 mm. pressure is about 5mm. Experiments were therefore made with 160

FIG. 4

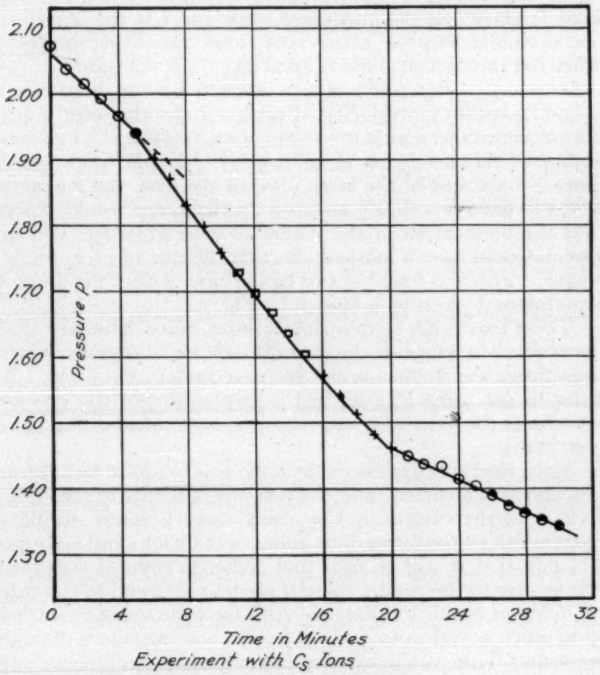


volt Cs ions at pressures of hydrogen ranging from 0.005 mm. to 0.044 mm. and the rate per unit current plotted against the average pressure during a run. The results, given in Fig. 4, are, unfortunately, not very accurate, but show that the effect persists to pressures at which the mean free path is long compared to 5 mm. and that the effect is roughly proportional to the pressure over this range. This would indicate that the effect is not due to secondary electrons.

In a second set of experiments a magnetic field of 350 gauss was placed on the tube in a direction parallel to the filament and perpendicular to the electric field. A secondary electron, starting from rest at the grid, and making no collisions, would travel along a cycloidal path coming back to the grid at a distance from the point at which it started given by $2\pi(E/H^2) \times (m/e)$ and traveling a distance toward the filament by

$2(F/H^2)(m/e)$. The maximum velocity it would obtain would be $2(F/H)$. In the experiments performed the magnetic field strength was 350 gauss, the electric field strength was 150 volts/cm. (45 volts in 3 mm.), and the pressure was about 0.023 to 0.015 mm. In the absence of collisions the maximum ve-

FIG. 5



Experiment with C_s Ions

○ No Voltage on; no Magnetic Field on.
\+ 45 Volts on; no Magnetic Field on.
□ 45 Volts on; Magnetic Field 350 Gauss
◓ No Voltage on; Magnetic Field 350 Gauss
To convert pressures to mm Hg multiply by 8.45×10^{-3}

locity obtained would correspond to 2.12 volts, the distance (perpendicular to both fields) between zeros of the cycloid would be 0.4 mm., the maximum excursion toward the filament 0.12 mm. and the total length of path from one zero of the cycloid to the next 0.492 mm. Now the mean free path of an electron at the pressure used is about 10mm. and the chance of making a collision in traversing one cycloidal loop is $e^{-l/\lambda}$, where λ is the mean free path and l is the length of one cycloidal loop, or about 0.05; and the chance of being scattered through an angle as great as 45° per collision is of the order of 0.1. If an electron were scattered through 45° when it had its maximum velocity, it might, under the most favorable circumstances pick up as much as 11 volts. The probability of this occurring is, as has been shown, quite small. Since, on account of the large mass of the ions, the magnetic field will have practically no effect on them, one would expect that the application of the magnetic field while the voltage was on would have a marked effect on the rate of pressure decrease. This was found not to be the case, within the limit of experimental error, as is shown by Fig. 5.

These two types of experiment show quite definitely that the observed change in the rate of decrease of pressure when the voltage was applied is not due to secondary electrons. In order to get some idea as to the mechanism of the process occurring, the following experiments and calculations have been made.

Since the rate of pressure decrease was found to be a linear function of the current and the pressure, the rate in mm/min. divided by the current and by the average pressure during a run was calculated for various voltages of Cs ions and is shown in Table II. It will be seen that although there is considerable scatter in the results there is no definite trend to the rate for different applied voltages. Although the voltages varied by as much as sixteenfold, the rates did not vary more than 50 per cent. This would seem to indicate that the process was not a collision of the first kind between the fast moving positive ions and the hydrogen molecules, since one would expect the rate to depend rather markedly on the voltage.

Another experiment in which two grids were used and the positive ions accelerated to the first grid and retarded between

the first and second grids was performed. The electrical conections are shown in Fig. 2. The filament, F, was made 90 volts positive with respect to the grid, G_1; and the grid, G_2, could be put at the same potential as G_1 or at any potential up

TABLE II.

Effect of Voltage.

A. *Li Ions*

Rate (mm./min. $\times 10^3$).	Current Amps. $\times 10^8$.	Pressure H_2 mm. Hg.	Average Rate $\left(\text{mm./min.} \times \frac{1}{\text{amp.}} \times \frac{1}{p} \times 10^{-1}\right)$.	Volts.
11.1	15	0.0177	4.19	160
13.5	16	0.015	5.60	80
8.7	12	0.013		
9.3	10	0.024	3.87	41
9.3	23	0.026	5.12	7

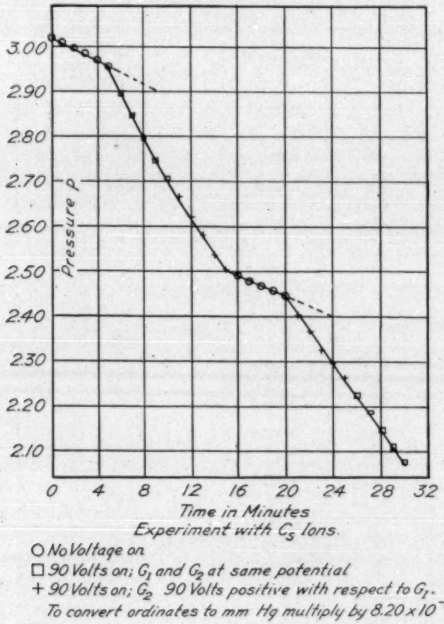
B. *Cs Ions*

Rate (mm./min. $\times 10^3$).	Current Amps. $\times 10^8$.	Pressure H_2 mm. Hg.	Average Rate $\left(\text{mm./min.} \times \frac{1}{\text{amp.}} \times \frac{1}{p} \times 10^{-1}\right)$.	Volts.
9.03	28	0.026		
9.75	28	0.026	1.37	320
9.86	26	0.0245		
12.0	30	0.0255		
11.5	30	0.020	1.75	240
11.3	21	0.033		
9.6	21	0.030	1.58	160
4.9	12	0.021		
2.8	12	0.019	1.59	40
2.18	6	0.029	1.25	20

to 90 volts positive with respect to G_1 by means of the battery B_2, and the slide wire potentiometer, P. The switch system S_1 to S_4 allowed currents to G_1 and G_2, or to G_2 alone to be read on the multimeter, M. The distance between F and G_1 was 2mm., and between G_1 and G_2 2 mm. The procedure was to accelerate ions from F to G_1, when G_2 and G_1 were connected together and measure the rate of decrease of pressure. Then

G_1 was made 90 volts positive with respect to G_2 (the potentiometer was adjusted until no current was flowing to G_2, and the voltage was read as 90 volts on a voltmeter), the experiment repeated. These runs were taken consecutively together with zero runs (no voltage applied between F and G_1).

FIG. 6.



Within the limits of error, no difference could be found between the two experiments, as is hown by Fig. 6. This shows that the effect is not dependent on the energy given to the positive ions and hence cannot be due to a collision of the first kind.

In order to see if the effect were due to some process occur-

ring on the filament when the voltage was applied, experiments were performed in which the filament was not coated with any positive ion emitter and the usual procedure carried out. No change in rate could be found when the voltage was applied. Another experiment, in which the filament was coated with the emitter but the potential was applied in the reverse direction so as to draw no positive ions from the filament to the grid (F 90 volts negative with respect to G_1 ; G_2 removed) showed no effect when the voltage was applied. Since previous experiments had shown that the effect was proportional to the current flowing between the filament and the grid, it is certain that the process is not occurring on the filament.

In order to test the purity of the hydrogen, the palladium tube was no longer used but tank hydrogen stored over sodium was admitted to the tube instead.[15] Experiments with this type of hydrogen showed the same effect, and to the same order of magnitude, as those with the hydrogen from the palladium tube. Thus the effect does not seem ro be due to impurities in the hydrogen.

In order to see if the Pirani gauge was working correctly, experiments were made in which pressures could be measured on the McLeod gauge as well as the Pirani. In order to do this, the experiment had to be run over a considerable length of time so as to obtain a pressure decrease large enough to read on the McLeod. Even at that the error in reading the McLeod was a considerable fraction of the measured pressure decrease. The experiments did show however, that the pressures as measured on the Pirani corresponded with those as read on the McLeod and that there was a greater pressure decrease when the voltage was applied than when it was not.

In order to obtain some further information as to the products of the process a test for "recondensible products" was made. Hydrogen was admitted to the tube, the liquid air level adjusted, and the pressure read on the McLeod gauge. The filament was then heated and allowed to run about 30 minutes with no voltage applied. The filament was then turned off, the system allowed to come to equilibrium and the pressure read. The liquid air was then removed for an hour,

[15] See A. C. G. Mitchell and R. G. Dickinson, *Jour. Amer. Chem. Soc.*, 49, 1478 (1927).

then replaced, and the final pressure read. The same experiment was repeated with 90 volts applied (Cs ions). From these results the precentage of the total decrease in pressure which was due to products recondensible in liquid air could be calculated. The results showed that with no voltage applied 27 per cent, and with the voltage applied 60 per cent of the products were recondensible. In the ideal case, when the filament was pure platinum with no coating and when there were no oxides of any kind in the tube, one would expect a decrease of pressure due to the thermal formation of atomic hydrogen on the filament and its subsequent condensation on the cold glass walls of the tube. This hydrogen should not be recondensible in liquid air. Similarly, if the process involved when an ion current were flowing was the formation of atomic hydrogen by collision of the first kind, the products should also be non-recondensible. On the other hand, if the decrease in pressure caused by the ions were due to the formation of some condensible product such as CsH, then a condensible portion of the products should be recondensible. No great weight should be attached to the experiment tried in this case, since there were considerable recondensible products formed when no voltage was applied. The experiment does seem to indicate, however, that more recondensible products are formed when the ions are flowing than when they are not.

SUMMARY OF RESULTS AND DISCUSSION.

Several outstanding results have been shown by the above experiments, viz.,

1. With hydrogen in the tube, and with liquid air surrounding the walls, the pressure of hydrogen was found to decrease more rapidly with the time when the voltage (from 15 to 320 volts) was applied, and Li and Cs ions were being accelerated away from the filament than when no voltage was applied.

2 The rate of decrease of pressure, with a given accelerating voltage, was found to increase with the ion current and with the average pressure of the hydrogen during the run.

3. The rate of decrease of pressure per unit current per unit pressure was found to be independent of the voltage.

4. It has been shown by two types of experiments that the effect is not due to secondary electrons.

The experiments show that the effect is probably not due to a dissociation of hydrogen by a collision of the first kind between a fast positive ion and a hydrogen molecule, since it is practically independent of the voltage applied to the tube and is not affected when a retarding potential is applied to the ions by a second grid. Since the process takes place in the gas phase (proportional to the ion current and the pressure) and is not due to secondary electrons, it is most probably due to the formation of an alkali hydride when the alkali ion comes in contact with the hydrogen molecule. This point of view is borne out by the experiments which go to show that there are more recondensible products formed when the voltage is on than when it is off.

It remains to discuss the differences between these experiments and those of Leipunsky and Scheckter.[11] In the first place they used a Kunsman catalyst emitter instead of the positive ion sources used in this experiment. They used a McLeod gauge to measure pressures whereas a Pirani gauge was used in these experiments. In other respects their arrangement was similar to the one used here. They found definite voltages at which effect began for the different ions used whereas no such effect was obtained here. Their rates of pressure decrease were of the order of (8×10^{-4} to 5×10^{-3} mm./min.), whereas those reported here are of the order of (1×10^{-4} to 5×10^{-4} mm./min.). From the dimensions of the tube as given in their paper, its gas volume (allowing something for the volume of the lead in stem) would be about 300 c.c. Allowing 100 c.c. for the volume of their McLeod gauge and connecting tubing, the total volume would come to about 400 c.c. This would seem to be a conservative estimate. Their currents (presumably measured from the filament to the first grid, although this is not quite clear) varied from 2×10^{-7} to 4×10^{-6} amps., whereas the current flowing between the filament and the first grid in the present experiments varied from 6×10^{-6} to 60×10^{-6} amps. and were consequently about fifteen to thirty times larger. Since the apparatus here used could detect smaller pressure changes than that of Leipunsky and Schechter and since the current flowing was considerably larger, it would be expected that an effect of the size reported by them would have been detectable in the present experiments.

It is perhaps unfair to criticize another's experiments without seeing the apparatus and one hesitates to do so, but there is one respect in which their experiments seem strange, namely that such large pressure decreases are obtained when such small current is used. If it is assumed that the current given in their data is that flowing from the filament to a reasonable sized grid, that the volume of their apparatus is about 400 c.c., and the average temperature as 300° K, (100° K, would do just as well since it only introduces a factor of 3 in the result), then from Fig. 4 of their article for 125 volt Li ions, one finds that there are about 1000 hydrogen molecules disappearing for every positive ion entering the tube, and from Fig. 6, for 270 volt K ions about 10 molecules per ion. This calculation is made by taking the difference of slopes of the curves when the voltage is on and when it is off, and calculating how many hydrogen molecules would have to disappear per second to account for the observed rate. Comparing this with the number of ions entering the tube per second from the current the above results are obtained. This seems somewhat surprising in view of the work of Glockler, Baxter, and Dalton which indicates that much less than one hydrogen molecule disappears per electron. From the figures given in Table II, or Fig. 4 of this paper, *one can easily calculate that there are from 0.01 to 0.5 molecules disappearing per ion*, which is itself rather large but not too large to be accounted for if the ions react chemically with the hydrogen.

It is conceivable that, in the experiments of Leipunsky and Schechter where the pressures were rather high and the mean free path of the ions short, a fast positive ion might lose a certain fraction of its energy in dissociating a hydrogen molecule and still retain enough to dissociate another etc., each time losing a fraction of its energy. If this were the case it would be difficult to see how a critical potential could exist; or if it did, to understand how the efficiency of dissociation could be so high very near the critical potential. The calculations for K ions with 270 volts show 10 molecules disappearing (within 10 volts of the critical potential 260 volts) per positive ion. From the above considerations it is hard to see how the effect observed by Leipunsky and Schechter can be due to dissociation by a collision of the first kind.

ANTIFOGGING AGENTS IN DEVELOPERS.

BY

A. P. H. TRIVELLI and E. C. JENSEN.*

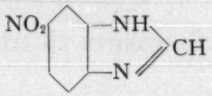
Introduction.—According to Eder,[3] C. Russell (Tannin Process, ed. 2, 1863) made the first recommendation for the addition of potassium bromide to a developer as a restrainer and as an antifogging agent. Potassium iodide was used in developers for the same reason in 1880 by Carey Lea,[5] and in 1882 by Székely.[12] A. H. Nietz [8] investigated the action of potassium citrate, potassium ferrocyanide, potassium nitrate, potassium oxalate and potassium sulfate on a hydroquinone developer. The fog for 12 minutes development was reduced in nearly all cases, but Nietz is inclined to attribute the effect of these salts, as distinguished from halide salts, principally to a physical effect on the gelatin and possibly to adsorption. Recently S. E. Sheppard and H. Hudson [11] found new antifogging agents in the thioanilides. They investigated amounts of one part in 100,000, 50,000, and 1,000 parts developer. After the introduction of a fog correction agent they found that in all cases the exposure scale also was largely altered. The densities in the toe of the curve were reduced and the maximum density and gamma cut down.

In U. S. Patent No. 1,696,830 P. Wulff patented a number of substances which, when added to a photographic emulsion, will diminish emulsion fog. Certain products obtained from the decomposition of proteins, for example, histidine (iminazolalanine), have this effect. Other products of this character include compounds of the iminazole series, such as nitrobenziminazole, thioglycollic acid, thiolactic acid, tetrazol, 2-6-dimercapto-4-keto-3, 5-diphenylthiopene and allyl aminothiobiazolsulfhydrate.

Our purpose was to investigate the antifogging action of these substances in a developer, instead of in the photographic

* Communication No. 440 from the Kodak Research Laboratories.

emulsion, and we chose the compound 6-nitrobenziminazole to start with. This has the structure:



Photographic Material and Development.—We restricted our investigation to the motion picture negative film of the Eastman Kodak Company.

Development was effected with hand brushing, according to W. Clark's method,[1] in a bath the temperature of which was controlled by thermostatically operated jackets. The temperature employed was 20° C. ± 0.1°.

The antifogging action in hydroquinone, pyrogallol, and p-aminophenol (hydrochloride) developers was investigated. These developers were compared with the same molecular equivalents and with the same mounts of sodium sulfite (Na_2SO_3) and sodium carbonate (Na_2CO_3) added. The formulas are given in Table I.

TABLE I.

Developing Formulas.

Developing Agent.	Weight of Developer.	Sodium Sulfite.	Sodium Carbonate.	Water to
Hydroquinone.....	5.5 gm.	50 gm.	50 gm.	1000 cc.
Pyrogallol.....	3.15 gm.	50 gm.	50 gm.	1000 cc.
p-aminophenol.....	7.275 gm.	50 gm.	50 gm.	1000 cc.

The sulfur dioxide content of the sodium sulfite was determined analytically and the actual amount calculated for an equivalent of 100 per cent. pure salt. The plates were developed for 3, 4, 6, 8, 10 and 12 minutes, and fixed in an acid fixing bath.

The investigations for the hydroquinone development were repeated five times and the results averaged. As the densities obtained varied so slightly in the repetitions, we regarded it as quite sufficient to repeat the investigations for the pyrogallol and for the p-aminophenol developer only three times. A Martens densitometer was used to read the densities.

The Sensitometer.—The exposures were made with a non-intermittent sensitometer with $\sqrt{2}$ ratio exposures of 20 steps. The light intensity was kept constant and the time of exposure was varied. The maximum time of exposure was 724 seconds. The light source was a 500 candle power standard monoplane tungsten filament lamp, operated at 89.0 volts and 4.05 amperes to give, at a color temperature of 2900° K., 0.0348 candle power intensity on the plate which was 101.6 cm. distant from a diaphragm covered with opal glass to act as a light scatterer when placed before the light source. To keep the light intensity as constant as possible a voltage regulator, which maintained the voltage constant to within one volt, was designed by A. L. Schoen of these Laboratories. A record of the voltage changes is shown in a previous publication by one of us [13] on the Herschel effect. In this work the deviations correspond with an intensity difference of less than 6 per cent. No filters were used.

Expression of Results.—Sheppard and Mees, also Nietz,[8] pointed out that fog is practically absent from the high densities, but increases as the image densities decrease and that an antifogging agent should restrain fog more than the image. The formulas for fog correction proposed by W. Meidinger [7] and R. B. Wilsey [14] are based on similar assumptions which could not be tested separately, but which are reasonable and acceptable. The investigations of H. A. Pritchard [9] showed that the Meidinger-Wilsey fog correction holds for fog, the density of which is not more than 0.51. P. Wulff claims that the fog-removing qualities of the organic compounds patented by him do not affect the densities of the image. This is something entirely new from what has been known or could be expected before. It was therefore of interest to investigate the image density depression of these substances in the developer to find whether they behave the same as when added to the photographic emulsion. Nietz [8] expresses the density depression by the downward shift of the intersection points of the straight line portion of the H. & D. curves for different times of development. Since we were often unable to determine this intersection point, we used another expression:

$$\text{Depression} = \frac{D_0 - D_a}{D_0} \times 100,$$

in which D_0 is the density obtained with the developer that did not contain the antifogging agent, and D_a is the density obtained with the same developer with the addition of the antifogging agent, under equivalent conditions of exposure and development.

By plotting the depression against log E we get the depression curves for different exposures. The depression curves for different times of development can easily be derived from the published curves.

According to Eder [1] a fog density of 0.2 is practically without any influence on the image. M. L. Dundon and J. I. Crabtree [2] also accept this density as the maximum limit of fog allowable in practice.

In the present investigation we compared the antifogging action of 6-nitrobenziminazole with that of potassium bromide and potassium iodide by trying to find the concentrations necessary in the hydroquinone developer to keep the fog, at 12 minutes development, at approximately the same value near, and at the same time below, the density 0.2. This was determined by a series of experiments with varying amounts of antifogging agent. The same concentrations of antifogging agents were then used in the pyrogallol and in the p-aminophenol developer.

Since a very large number of data was collected in this way it is out of the question to publish complete tables. It may be suggested that adequate comparison can be effected by:

(*a*) The graphic exposition of the families of H. & D. curves from 3 to 12 minutes development;

(*b*) The graphic comparison of the depression curves for different exposures;

(*c*) The graphic exposition and comparison of the second derivatives for three different times of development (3, 6, and 12 minutes) from which may be secured numerical comparisons of contrasts;

(*d*) The graphic exposition of the growth of fog with time of development;

(*e*) The graphic exposition of the resolving power for different times of development and for different densities.

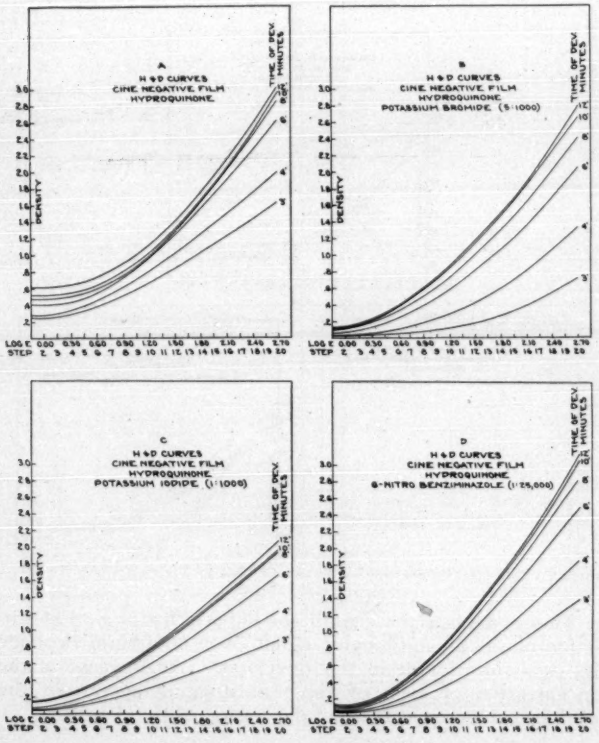
These data are sufficient for the deduction of other functions and constants.

The graphic exposition of the growth of gamma with time of development is omitted since, in most cases, no straight line portion of the curve could be found.

DISCUSSION OF THE RESULTS.

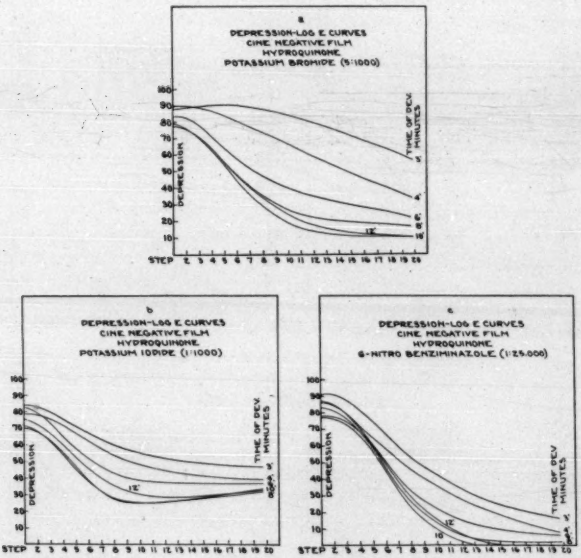
Motion Picture Negative Film with Hydroquinone Developer.—In Fig. 1 are given the H. & D. curves obtained with the hydroquinone developer alone, and in combination with

FIG. 1.



potassium bromide, potassium iodide, and 6-nitrobenziminazole in various concentrations (as shown in the figures). These concentrations of the three antifogging agents give about the same fog diminishing in the unexposed parts of the film as is shown in Fig. 4. All other comparisons between the antifogging action of these substances refer only to these concentrations. In *C* in Fig. 1 is noticeable a slight increase of the threshold value, probably due to the Lainer effect.

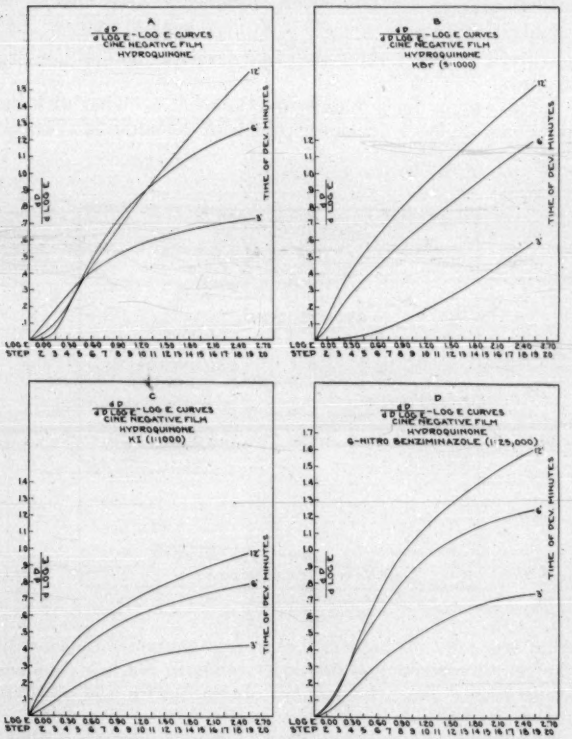
FIG. 2.



These curves show clearly the superiority of 6-nitrobenziminazole as an antifogging agent over potassium bromide and potassium iodide in this developer. An increase of the concentration of each of these antifogging agents in the developer gives an increase of depression of the densities.

In Fig. 2 are given the depression-log E curves calculated from the data plotted in Fig. 1. The fog removing power of 6-nitrobenziminazole is slightly greater than that of potassium

FIG. 3.



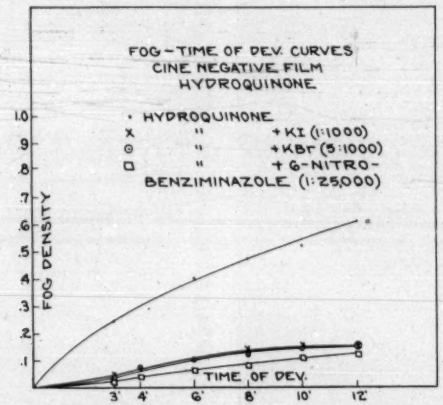
bromide and potassium iodide as far as the above concentrations have been applied. Comparing potassium bromide with potassium iodide, we see that for the shorter times of de-

velopment the depression is greater for potassium bromide than for potassium iodide, but that for the longer times of development the depression is less for potassium bromide than for potassium iodide.

The $\frac{dD}{d \log E} - \log E$ curves representing the change of contrast with exposure for 3, 6, and 12 minutes development are given in Fig. 3, using the same developer and antifogging agents.

The gamma for 3 minutes development is higher with the pure hydroquinone developer than with the addition of potas-

FIG. 4.

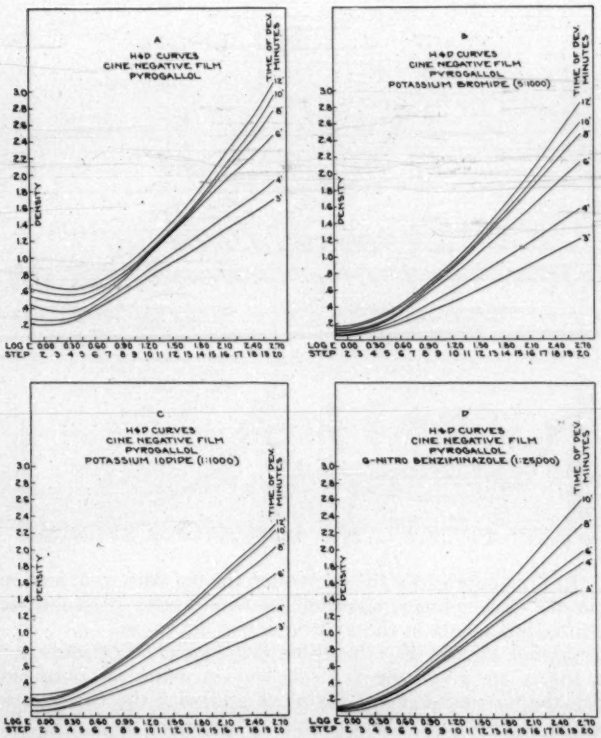


sium bromide. At higher exposures the gamma obtained with the pure hydroquinone developer tends to reach a constant value sooner than the gamma obtained with the addition potassium bromide in the developer.

For 6 minutes development the gamma curves are about the same except that at the lower exposures the gamma obtained with potassium bromide in the developer is higher. For 12 minutes development this difference becomes even

more pronounced and extends to higher exposures. Potassium iodide added to the developer gives lower maximum gamma values, as is shown by *C* in Fig. 3. Higher gammas are only obtained at the lower densities. In *D* it is seen that with 3 minutes development, using 6-nitrobenziminazole, the gamma is smaller over nearly the whole range of exposures than the gamma obtained with the pure hydroquinone developer,

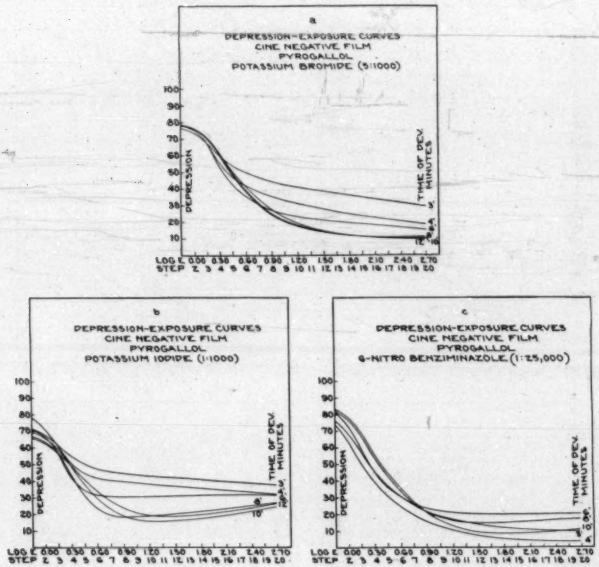
FIG. 5.



and with 6 and 12 minutes development the gamma is higher over nearly the whole range of exposures.

Comparing the gammas of *B* and *D* of Fig. 3 with each other, potassium bromide, at the lower exposures, gives a higher gamma than 6-nitrobenziminazole added to the hydroquinone developer.

FIG. 6.



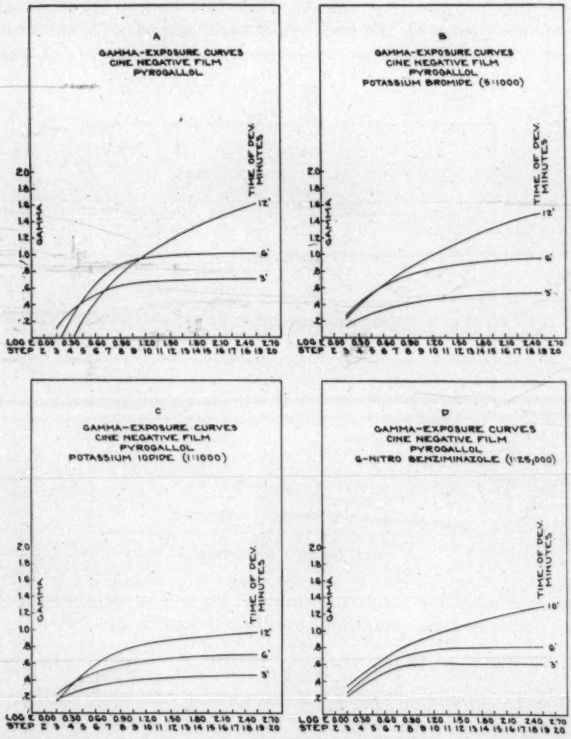
In Fig. 4 are given the curves for the fog with 3, 6, and 12 minutes development, showing the total action of the three antifogging agents in the hydroquinone developer.

Motion Picture Negative Film with Pyrogallol Developer.—In Fig. 5 are given the H. & D. curves which we obtained with the pyrogallol developer alone and with the three antifogging agents at the same concentrations as used with the

hydroquinone developer, which allows for comparison between them.

The lower densities obtained with the pure pyrogallol developer show fog destruction. Addition of the antifogging

FIG. 7.

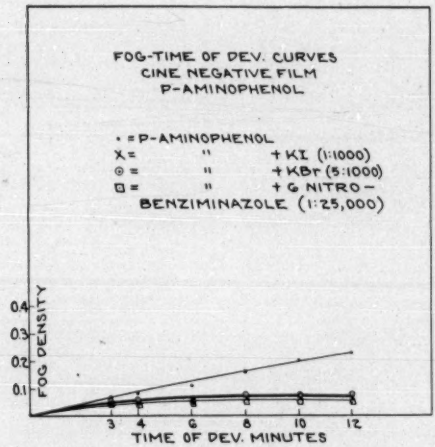


agent to the developer removes this effect. Neither the hydroquinone, nor the p-aminophenol developer shows this

reversal. It does not seem to be related to solarization because addition of potassium bromide to the developer does not remove solarization. C. E. K. Mees and C. W. Piper [6] call this a reversal by sulfite, the strip being similar in appearance to that obtained with thiocarbamide.

In Fig. 6 are given the depression-log E curves calculated from the data plotted in Fig. 5. Less difference in fog removal is shown than with the hydroquinone developer. It is, however, noticeable that 6-nitrobenziminazole, with about the

FIG. 8.

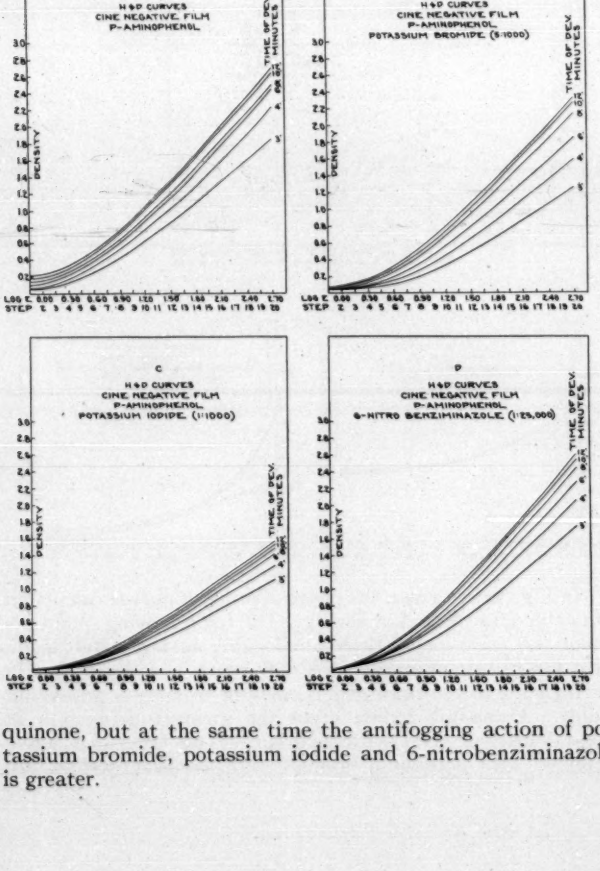


same average fog removing quality, gives less depression of the densities than potassium bromide or potassium iodide.

In Fig. 7 are drawn the $\frac{dD}{d \log E}$ – log E curves from the H. & D. curves of Fig. 5. The gammas obtained with the pyrogallol developer are lower than those obtained with the hydroquinone developer. With the exception of a few lower exposures the general behavior is about the same as with the hydroquinone developer.

In Fig. 8 are given the increase of fog with increasing time of development. In comparison with Fig. 4 we see that pyrogallol gives more fog with longer development than hydro-

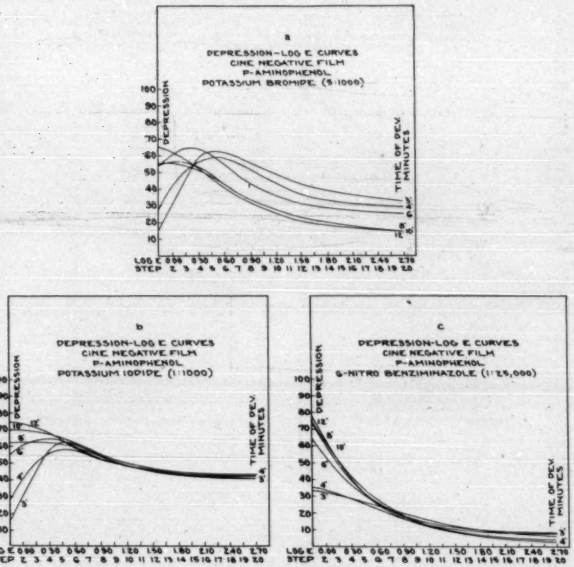
FIG. 9.



quinone, but at the same time the antifogging action of potassium bromide, potassium iodide and 6-nitrobenziminazole is greater.

Motion Picture Negative Film with p-Aminophenol Developer.—In Fig. 9 are given the H. & D. curves obtained with p-aminophenol developer alone and with potassium bromide, potassium iodide, and 6-nitrobenziminazole in the same concentrations as used before.

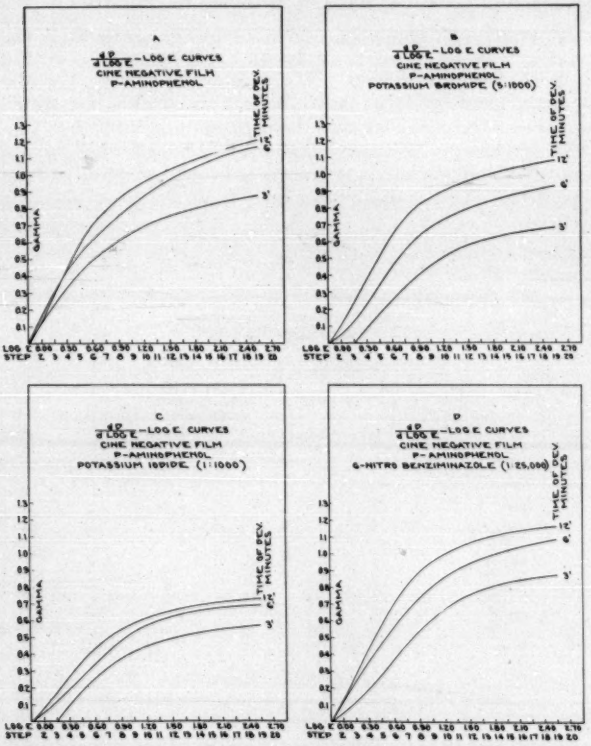
FIG. 10.



In Fig. 10 are given the depression-log *E* curves calculated from the data plotted in Fig. 9. The fog removing quality is about the same for all three antifogging agents. But again, with 6-nitrobenziminazole, we find much less depression in the higher densities than with potassium bromide or potassium iodide. Potassium iodide gives the greatest depression in these densities. The depression curves, for lower exposures, obtained with potassium bromide and potassium iodide, are

quite different in shape from those obtained for the pure developer and with 6-nitrobenziminazole. On account of the small differences in the low fog densities in this exposure region

FIG. 11.

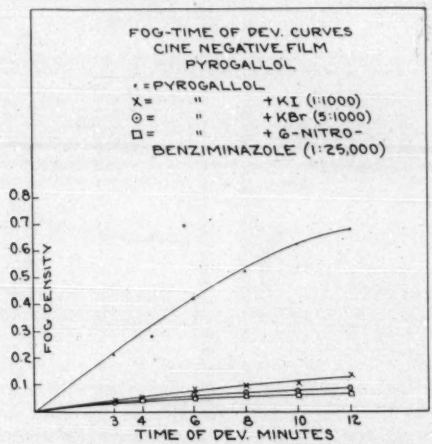


with the p-aminophenol developer the data obtained are less reliable, so that no definite significance can be attached to this deviation from the ordinary shape of the depression curves

without further investigation. The sensitometric strips, which were developed with 6-nitrobenziminazole for 12 minutes, show very little difference in image densities compared with the densities developed with the pure p-aminophenol developer. At first sight the impression is created that only the fog is entirely removed without changing the densities.

In Fig. 11 are drawn the $\frac{dD}{d \log E} - \log E$ curves from the H. & D. curves of Fig. 9. The gammas obtained with the p-aminophenol developer show effects of the antifogging agents similar to those obtained with the hydroquinone and the pyrogallol developers. Potassium iodide gives the greatest gamma depression and 6-nitrobenziminazole gives the smallest gamma depression. This could be expected from the depression-log E curves.

FIG. 12.



In Fig. 12 is given the increase of fog with different times of development. The fog increase is for the antifogging agents near the limit of observation.

Exposure for the Determination of the Resolving Power.—With G. Silberstein we investigated the influence of these antifogging agents on the resolving power. The exposures on each strip range from 1/128 second to 8 seconds, each exposure being twice the preceding one. Sandvik's parallel line test object [10] of a contrast of 1000 between the background and the transparent slits was used. This was illuminated by a tungsten filament lamp, the light from which was rendered diffuse by being passed through a sheet of flashed opal glass.

The camera was focused to give the maximum resolving power on the motion picture positive film, the same focus being used for the motion picture negative exposures.

The resolving power is expressed as the number of lines per millimeter resolved, and is plotted against the image density for each exposure step.

Discussion of the Results.—In Table II is given the maximum resolving power ($R.P._{max.}$) obtained with the different developers and the additional antifogging agents, together

TABLE II.

$R.P._{max.}$ for Motion Picture Negative Film.

Developer.	R.P.max. for			*D* at R.P.max. for		
	3′	6′	12′	3′	6′	12′
Hydroquinone	41.5	35.5	35.5	1.6	2.4	3.0
" +KBr	48	43	38	0.8	1.4	1.2
" +KI	35	34.5	33	0.7	1.3	1.6
" +n.b.[1]	46	40	38	1.1	1.6	1.6
Pyrogallol	36.5	40	39.5	1.4	2.0	2.6
" +KBr	44	40	39.5	1.1	1.3	1.6
" +KI	36.5	36	35	1.0	1.2	2.0
" +n.b.	40	40	40	1.0	1.5	1.9
p-Aminophenol	40	39.5	38.5	1.7	1.7	1.7
" +KBr	50	48.5	42	1.0	1.2	1.4
" +KI	36.5	34	32	1.0	1.2	1.4
" +n.b.	46	39.5	35.5	0.8	1.8	1.8

with the density *D* at which the maximum resolving power occurs. We see in general a tendency to shift the maximum resolving power with time of development to higher image densities. Potassium bromide added to the developer gives, at 3 minutes development, the highest resolving power.

[1] n.b. = 6-nitrobenziminazole.

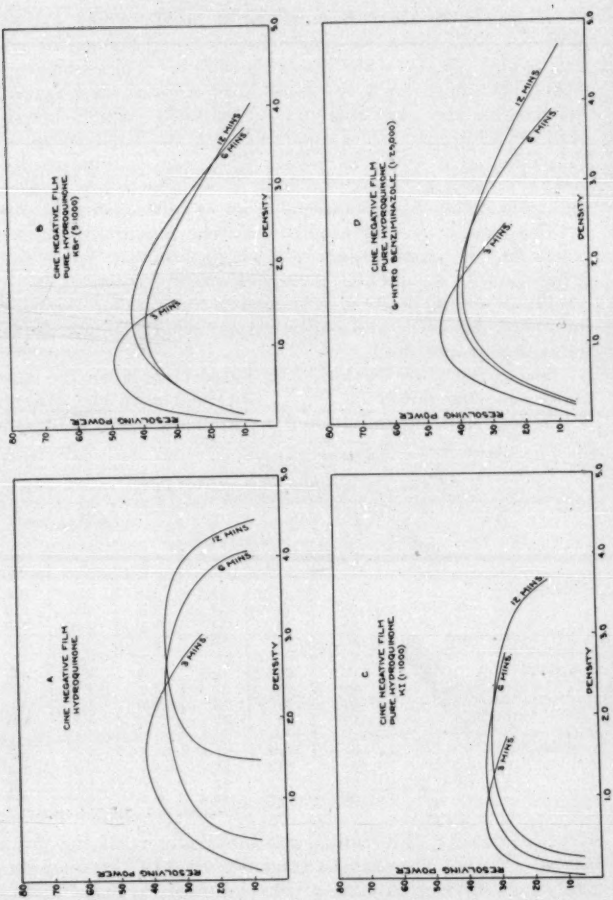

A
CINE NEGATIVE FILM
HYDROQUINONE
B
CINE NEGATIVE FILM
PURE HYDROQUINONE
KBr (1:1000)
C
CINE NEGATIVE FILM
PURE HYDROQUINONE
KI (1:1000)
D
CINE NEGATIVE FILM
PURE HYDROQUINONE
6-NITRO BENZIMIDAZOLE (1:24,000)
3 MINS
6 MINS
12 MINS
DENSITY
RESOLVING POWER

FIG. 13.

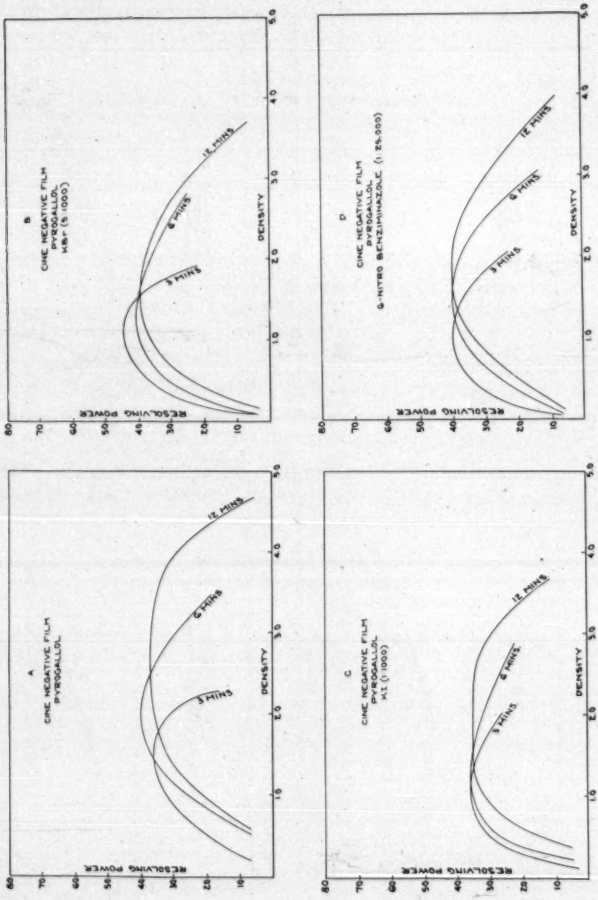

A
CINE NEGATIVE FILM
PYROGALLOL
B
CINE NEGATIVE FILM
PYROGALLOL
MET (5-1000)
C
CINE NEGATIVE FILM
PYROGALLOL
MET (1-1000)
D
CINE NEGATIVE FILM
PYROGALLOL
6-NITRO BENZIMINAZOLE (1-25,000)
RESOLVING POWER
DENSITY
3 MINS
6 MINS
12 MINS

FIG. 14.

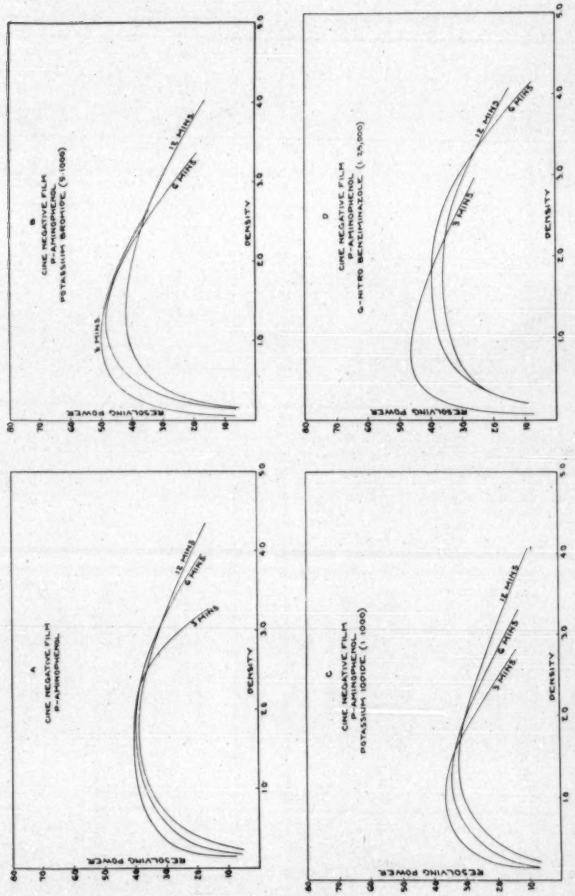

A
CINE NEGATIVE FILM
P-AMINOPHENOL
B
CINE NEGATIVE FILM
P-AMINOPHENOL
POTASSIUM BROMIDE (1:1000)
C
CINE NEGATIVE FILM
P-AMINOPHENOL
POTASSIUM IODIDE (1:1000)
D
CINE NEGATIVE FILM
P-AMINOPHENOL
6-NITRO BENZIMIDAZOLE (1:15,000)
15 MINS.
6 MINS.
3 MINS.
RESOLVING POWER
DENSITY

FIG. 15.

For the purpose of comparing the three developers the highest values were averaged, with the following results:

TABLE III.

Average R.P.$_{max.}$ for Motion Picture Negative Film.

Developer.	A.	B.	C.	D.
Hydroquinone	37.5	43	34	41
Pyrogallol	39	41	34	40
p-Aminophenol	39	47	34	40

Interesting differences in results when different antifogging agents were used can readily be seen by study of Figs. 13 to 15 in connection with Tables 2 to 4. In the figures, and in Table III, are the data obtained with:

the pure developer	represented by *A*
the pure developer and potassium bromide	represented by *B*
the pure developer and potassium iodide	represented by *C*
the pure developer and 6-nitrobenziminazole	represented by *D*

From the figures and table we see that the highest maximum resolving power in general is produced by the p-aminophenol

TABLE IV.

Density at a R.P. = 10.

Motion Picture Negative Film.

Time of Development	A	B	C	D
	Hydroquinone			
3′	0.14	0.07	0.04	0.08
6′	0.50	0.14	0.16	0.21
12′	1.48	0.14	0.25	0.24
	Pyrogallol			
3′	0.27	0.09	0.18	0.05
6′	0.59	0.10	0.27	0.12
12′	0.66	0.18	0.43	0.21
	p-Aminophenol			
3′	0.17	0.06	0.15	0.07
6′	0.19	0.25	0.18	0.21
12′	0.27	0.27	0.26	0.21

with potassium bromide, with 6-nitrobenziminazole next highest. The latter, with pyrogallol, gives better results in general than potassium bromide.

We investigated the resolving power at the lower exposure end of the curves and determined at which density the resolving power had reached a value of 10. The data are given in Table IV, the letters *A*, *B*, *C*, and *D* bearing the same significance here as in the figures and in Table III.

A tendency of the resolving power to diminish with increasing time of development may be observed. In the lowest exposure end of the curve potassium bromide does not give the best results, when used in P. A. P. developer. A noticeable fact is the great improvement of the resolving power by all the antifogging agents in the toe of the H. & D. curve, a fact that was already known in common practice. With the p-aminophenol developer 6-nitrobenziminazole did not give as good results as bromide or iodide, the reason apparently being that the concentration employed (1 part 6-nitrobenziminazole to 25,000 parts p-aminophenol developer) is too high and tends to destroy the lowest densities while depressing the fog in spite of the fact that it gives less depression with the higher densities than potassium bromide and potassium iodide. The fact that 6-nitrobenziminazole does not produce as high a maximum resolving power as does potassium bromide (except in the pyrogallol developer) would also indicate that the concentration of the former in the developer was too high.

CONCLUSION.

(1) As an antifogging agent in the hydroquinone, the pyrogallol and the p-aminophenol developers, 6-nitrobenziminazole is superior to potassium bromide and potassium iodide, giving the least depression of the image densities with the same fog removing quality. With the p-aminophenol developer conditions were approached such that the fog was removed up to 12 minutes development at 20° C. without change of the image densities.

(2) The investigation with G. Silberstein leads to the conclusion that with negative film 6-nitrobenziminazole in general does not give as good results as potassium bromide. There is reason to hope, however, that 6-nitrobenziminazole

if used in still weaker concentrations in the developer will give as good results over the complete density range as those given by bromide. This possibility is to be investigated in the near future.

We cordially extend our acknowledgment to Mr. George Silberstein for the determinations of the resolving power made by him.

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A New Force Exerted by Light on Matter. V. POSPISIL. (*Phys. Zeit.*, Jan. 15, 1930.) The author has found that strong light absorbed by small particles in suspension increases their Brownian movements. He has already published two papers on this subject. In general the process of experiment consisted in selecting a suspended particle in the field of the microscope and in following and recording its displacements under changes of illumination. No less than 17,000 such displacements were noted and form the basis of the conclusions drawn. The liquids in which the Brownian movements were watched were water, solutions of sugar in water and chloroform. The suspended particles were generally soot. The temperature of the liquid under experiment was obtained by replacing the cell containing it by an exactly equal cell subject to the same influences, the temperature of which was given by a thermoelement. Corrections were made for changes in temperature and viscosity caused by irradiation. In all cases the mean square value of the displacements of a particle under illumination was greater than in weak diffuse light. The increase runs up to 43.5 per cent. To a rough approximation the per cent. of increase is in proportion to the reciprocal of the viscosity, being two or three times as great for water as for aqueous sugar solutions.

By trying different sources of light it became clear that those sources are more effective that emit radiation rich in the middle and shorter wave-lengths. The light from a mercury arc produced an increase of 43.5 per cent. whereas an Osram incandescent lamp is credited with only 22.5 per cent., though the intensity of the radiation from the latter was about 1,000 that of the former. It was further found that strong infra-red light acts to reduce the effect being produced by optical illumination. In both faint and strong light all directions are equally probable for displacements in the plane perpendicular to direction of observation. This no longer holds when plane polarized light is used. With this the displacements in the plane of polarization are distinctly larger than in the plane at right angles to it.

Experiments were carried out with particles ranging in diameter from .001 to .0004 mm. In optical light the larger particles showed a smaller disturbance of their motions.

In a second paper in the same journal Pospisil endeavors to account for the disturbance caused by the absorption of radiation by the suspended particles. The effects of the momentum of a light particle, of temperature gradients, of recoil of molecules of liquid warmed by the particle and of the recoil of photo-electrons issuing from the particles, all are rejected as furnishing an adequate effect and the phenomenon is left without explanation to await further examination. G. F. S.

INSTRUMENT FOR DETECTING METALLIC BODIES BURIED IN THE EARTH.

BY

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SUMMARY.

This paper gives a description of a new instrument recently developed by the National Advisory Committee for Aeronautics at the Langley Memorial Aeronautical Laboratory. The instrument was made for the immediate purpose of locating unexploded bombs which were known to have been dropped from airplanes at targets in close proximity of the site of the new Seaplane Towing Channel at Langley Field, Virginia. The new "detector" successfully located a number of bombs buried on and near the projected site. It is of a simple design and requires no skilled operators.

The author gives a brief theoretical survey of the general nature of the difficulties encountered in the design of sensitive detectors of this type. He points, in particular, to the importance of avoiding capacity and resistance effects, and outlines other essential factors contributing to the success of the new detector.

INTRODUCTION.

This work was undertaken by the National Advisory Committee for Aeronautics for the purpose of detecting and removing unexploded bombs from the site to be occupied by the Seaplane Towing Channel. It was known that the projected channel was to pass through an area near one which had been used for bombing practice, and the Committee desired to take every precaution in order to avoid dangerous explosions of live bombs during the construction.

The problem was to obtain some instrument which could do the work; that is, which could locate the bombs that might be buried in the area referred to. It was desired that a layer of several feet in depth might be searched at one time.

While some experimental work has been done along such lines, no information of successful bomb detection is available, or has come to the author's notice.

An inquiry by the Committee at the Bureau of Mines resulted in an investigation of the practicability of employing an ultra-sensitive magnetometer. The scheme was, however, abandoned. It was found that the method was not sufficiently convenient for the purpose, and that in any case, the amount of labor involved would be prohibitive. Five minutes were required at each setup of the instrument, covering an area of possibly not more than from 10 to 15 square feet of the surroundings, and highly trained operators were required for such work.

A similar inquiry at the Bureau of Standards disclosed the fact that some attention had been paid to this question during the war. A brief résumé of work relating to this subject is published in the "War Works" of the Bureau. No record of instruments employed for actual field use is found, however.

The Bureau recommended the use of a Maxwell or Andersen bridge in connection with a large "search coil" of several henrys as being possibly the most sensitive method. The Committee tried this scheme:

A coil 8 feet in diameter containing 1300 turns of No. 21 wire was made up and inserted in a Maxwell bridge circuit.[1] The result was negative. It was found that the self-capacity of the coil, being of the order of 0.01–0.02 microfarad, was entirely too large to permit even an approach to the balancing of the bridge without extraordinary means. Being pressed for time, the Committee was forced to abandon the attempt, having in the meantime succeeded in developing an instrument of its own, which type in the following will be referred to as the N. A. C. A. Bomb Detector.

While it is too early to claim any absolute superiority of this instrument or of the principles involved as compared with others, the Committee feels, on the basis of the practical results obtained and the great simplicity of the scheme, that a description of this device might be of benefit to others. Several bombs were located after the first day's work, one of them

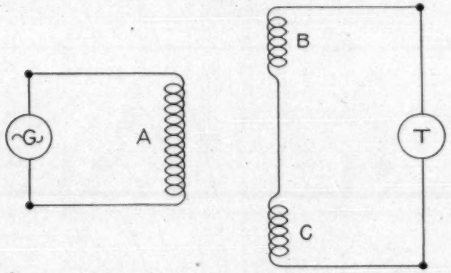
[1] For description of this circuit see, for instance, "Dictionary of Applied Physics," Vol. II, "Electricity," page 436, etc.

being a live T.N.T. 17 pounder buried with its center of gravity approximately 2 feet below the surface. While the exploded bombs all were found in holes of considerable size, no visible evidence disclosed the presence of the unexploded ones. The above bomb was set off on the following day, and a systematic scanning of the entire field taken up as a result. This work is completed at the writing of this article.

DESCRIPTION OF THE N. A. C. A. INSTRUMENT.

The diagram of the electrical system of the N. A. C. A. instrument is shown in Fig. 1. *G* is a high frequency generator working on a power coil *A*. *T* is a telephone supplied from the two opposing pick-up coils, *B* and *C*.

FIG. 1.



The three coils, *A*, *B*, and *C* are mounted symmetrically on a cylindrical wooden frame. A comparatively loose magnetic interlinking of the central power coil *A* and the two opposing pick-up coils, *B* and *C*, is employed. See Fig. 2. This feature is essential for the instrument and the reason for its importance is given in the following. The pick-up coils contain each 100 turns of No. 21 wire, while the power coil has about 75 turns of the same size. The coils are wound in a single layer in order to keep the self-capacity down.

The generator *G* supplied 500-cycle per sec. current, which frequency is probably the most satisfactory for the purpose. It was capable of supplying about 6 amperes at 110 volts.

The power coil was attached to this source of current by means of a step-up transformer.

The telephone T was specially designed to suit the low impedance of the pick-up coils. It consisted of a sensitive

FIG. 2.

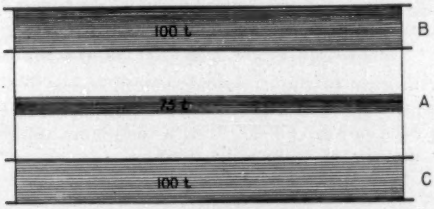


FIG. 3.

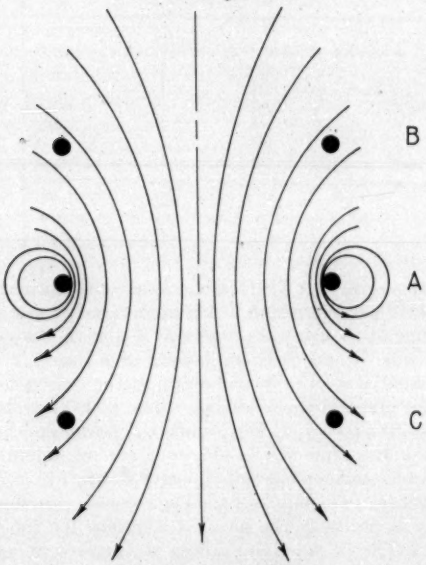


Diagram showing field in balance.

Baldwin head set rewound with about 100 turns of heavy wire. The operation of the instrument will be readily understood from the simple wiring diagram in Fig. 1. The "power" coil *A* supplies a strong alternating field. The electromotive forces set up in the two pick-up coils are in opposition to each

FIG. 4.

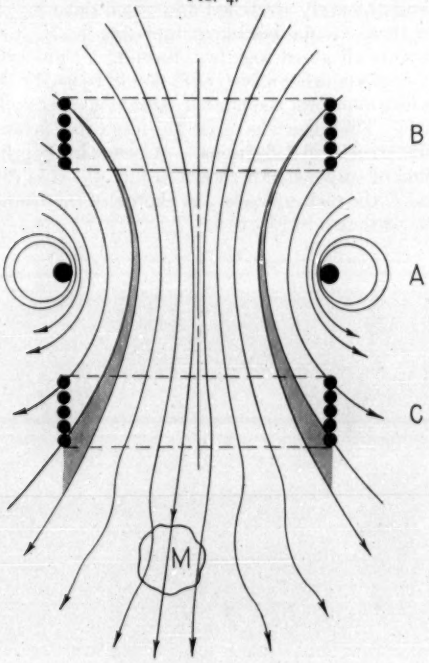


Diagram showing distortion of field due to introduction of magnetic material at M. The shaded portion is "audible."

other, so that only the difference appears across the telephone. In balanced condition *B* should be an "image" of *A* with respect to the central coil. When this image is perfect there is no current through the telephone. This condition is indicated in Fig. 3. If the instrument is brought close to a piece of

magnetic material, like a bomb, a distortion of the magnetic field lines takes place. This condition is shown exaggerated in Fig. 4. The shaded portion indicates the difference in magnetic flux through the two telephone coils *B* and *C*. This difference is consequently "audible" in the telephone. Care should be taken that the wooden frame be extremely rigid and the coils securely stretched and glued onto it. For this reason the three circular bobbins comprising the N. A. C. A. instrument were all glued together, forming a rigid cylindrical body. No nails or other metal parts should be used. To avoid local bending moments the handles were not directly fastened to the body. The latter was carried by four ropes fastened so as to minimize structural deflections. A "close up" of the design and method of suspension is shown in Fig. 5. *A* is the power coil, *B* and *C* the pick-up coils. A sketch of the frame giving dimensions is shown in Fig. 6.

FIG. 5.

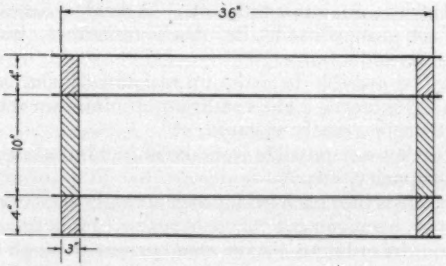
N. A. C. A. bomb detector.

It was on basis of experience gathered from a preliminary design shown in Fig. 13 that the above method of construction was adopted. The picture shown in Fig. 13 reproduces only what we have been referring to as the "power" coil, while

the two pick-up coils had been removed before the photograph was taken.

This method of fastening the handles proved to be entirely unsatisfactory. The instrument was thrown greatly out of balance on picking it up by the handles.

FIG. 6.



This larger coil of 5 1/2 feet in diameter was also somewhat too heavy for convenient operation.

DISCUSSION OF METHODS OF DETECTION.

It is hoped that the following analysis of the general difficulties involved in the design of an instrument of the present kind will serve to promote its further development.

Looking for some means of detecting a piece of iron, or other metals, several feet below the surface of the earth, the only conspicuous property of such material as distinguished from that of the soil is its effect on a magnetic field.

If a coil of a diameter which is comparable in magnitude to the depth desired is brought close to the ground above the buried metal, a definite change in the value of the self-inductance of the coil takes place. The power factor is also changed, due to the eddy currents set up within the metal. The change looked for is, however, so small (say 1 : 10,000) that any use of measuring instruments to indicate directly changes in current consumption or power factor of the coil is out of question.

The next step is to employ bridges as usually employed for

such purposes. The Maxwell bridge and the Hughes bridge will both serve the purpose.

The bridge systems are, however, apparently too complicated in adjustment. It must be appreciated that the value of the bridge as indicator depends entirely on the possibility of obtaining "zero" adjustment of the telephone or bridge current. This is not so easy. The question of securing absolute silence is an interesting one. A bridge contains four arms. Each arm contains its ohmic resistance, its capacitance, and its inductance.

It may be possible to make up resistances comparatively free from inductance. The condition of minimum self-capacity may then be greatly violated, etc.

It is further not possible to make inductances entirely free from self-capacity, etc.

The result is that each bridge arm actually is composed of a complicated arrangement of resistances, inductances, and capacities. In order to obtain zero current through the telephone, it is in consequence necessary to balance for ohmic resistance, for capacity, and for inductance. How this may be done is in itself a difficult mathematical and experimental problem, subject only to approximate solutions and trials. It may be done in the laboratory for a single frequency. If the imposed frequency is slightly changed, a new adjustment is, however, in general, necessary.

The problem is not so serious when it comes to measurement of comparatively large changes of say 1:500 or more in the value of the unknown inductance, particularly if a source of power of a wave-form as free as possible from harmonics is employed.

When it comes to greater accuracy, slight changes in the ohmic resistances due to temperature effects, small changes in the desired, or undesired, capacities and inductances associated with each arm of the bridge, are detrimental to the sensitivity of the method. The adjustment for silence is as mentioned only possible for one frequency which, of course, may be the most pronounced note, while all other disturbances, in general, pass through the telephone. Unavoidable changes in frequency, wave form and intensity of the source continuously tend to upset the balance. With a powerful source of

power which ought to be employed, the ear may not even be able to detect any definite minimum.

This strict requirement of an absolute "zero" current in the telephone at balance is brought about by the fact that the ear is able to distinguish between sounds of intensity 0 and 1, while no difference may be detected between sounds of intensity 10 and 11.

A great step forward in the direction of increased sensitivity is shown in the scheme disclosed in Drawing No. 583B of the Bureau of Standards, termed "Induction Balance for Detecting Metallic Bodies" and dated June 16, 1922. This type makes use of two transformers each in the form of coils of 5 feet in diameter. The system is shown in Fig. 9. It employs a separate variable mutual inductor and variable and fixed resistances for adjustments. Possibly the greatest objection to the scheme is the apparent close coupling between the primary and the secondary of the two transformers employed. This brings about considerable charging current between the two windings, apparently necessitating some means of compensation of the "out-of-phase" currents as evidenced by the interposed variable resistance.

A French article by M. C. Gutton entitled "Sur une Balance destinée a la Recherche des Obus enterrés dans les Terrains a mettre en Culture," published in *Comptes Rendus*, Volume 161, page 71, 1915, describes an instrument which is similar, with the exception that the resistances *AB* and *CD* of Fig. 9 are omitted. Mr. Gutton experiences, however, apparently the same trouble of not being able to obtain perfect silence, due to the same close coupling between primary and secondary of the two transformers, and probably also due to self-capacity of the coils. He avoids the trouble by cutting the power, as may be seen from the following quotation:

"La expérience m'a montré que, dans le large limites, cette sensibilité dépend du trembleur, qui doit donner un son musical aigu et avoir une trés petite amplitude d'oscillation." [2]

Both the French paper and the Bureau of Standards description given in the "War Works" of the Bureau indicate that primary and secondary of each transformer are wound

[2] "The sensitivity depends on the vibrator which should produce a distinct musical sound of a very small amplitude."

on common bobbins with the distance between the inside edges of the two coils equal to 1 centimeter.

The N. A. C. A. instrument employs only a single power coil which may be termed a primary with two opposing pick-up coils, both of which are in loose magnetic coupling with the former.

Since there is only a negligible change taking place in the force lines running close around the power coil when a piece of metal is introduced in the field, it will be seen that practically all of the change of the field is imposed on the pick-up coils. In fact, the system seems to be superior for the purpose for which it is intended, since the disturbing effect of a small piece of iron on the surface is almost avoided, while the full effect of a larger piece at a great distance is retained.

We will now indicate numerically the importance of avoiding the capacity effects between power and pick-up coils by a reference to the Bureau of Standards induction balance shown systematically in Fig. 9. With P_1 and S_1 5 feet in diameter and 1 centimeter apart and, for instance, wound with multiple layers, the capacity amounts to the order of 30 centimeters e.s.u. With a voltage of 300 and a frequency of 500 the current leaking across and going partly through the phones is of the order of some hundredths of a milliampere.

Now the 500 cycle current usually contains higher harmonics, and the condition is further aggravated, in particular, because the 1,000 and the 2,000 cycles are more "audible" than the fundamental, and the receiver, in addition, may "peak" at high frequencies.

It is evident that, as this charging current is avoided the sensitivity is increased, and the question of balancing greatly simplified. The N. A. C. A. Bomb Detector could be brought to perfect silence by means of one adjustment. In fact, the balancing was so good that the operator had a tendency to "forget" the note and actually preferred to keep the instrument slightly out of balance. The self-capacity of the pick-up coils is brought down to a negligible nagnitude by employing a single layer coil. The instrument is thus not sensitive to changes in generator frequency nor to irregularities in wave form. This simplifies the requirement as to the source of power.

The instrument is independent also of temperature differences and temperature changes.

The practical value of this latter fact can only be appreciated if one attempts to design bridge circuits having the same characteristics.

The instrument was adjusted by changing the position of the power coil or, in fact, by touching slightly *one* of its 75 turns, which acted as a very good vernier. The stability of the instrument was, however, so nearly perfect that on starting out for field work new adjustment was not always needed.

PRACTICAL CONSIDERATIONS REGARDING THE DESIGN OF THE N. A. C. A. TYPE.

The design of this instrument is so simple that anybody having at his disposal a small high frequency generator and a telephone receiver will have no difficulty in making up such a device. A few points to be taken in consideration will be brought up briefly in the following:

The N. A. C. A. Detector was operated satisfactorily in the field with one ampere through the power coil. If there is sufficient power available, there is no reason why the number of ampere turns should not be increased. The sensitivity of the instrument increases noticeably with the number of ampere turns employed in the primary. This is due to the fact, as brought out before, that perfect balancing is approached quite closely with this instrument.

The problem is thus to make up the power coil with as few turns as the available generator will permit.

The instrument should, for the sake of safety, not be operated in the field with more than a few hundred volts on the power coil; preferably with much less, depending on the necessary length of the supply line.

Regarding the question of "power" actually required for the instrument, it will be observed that, since no resistances form part of the circuit, the only energy consumption taking place is theoretically occuring in the telephone membrane.

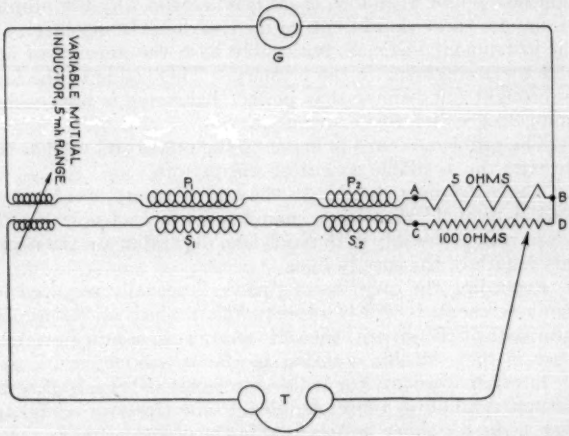
In other words, if the coils were made of wire having no resistance, all of the power supplied to the Detector would appear in the telephone and as heat in the object.

Considerations of weight will, however, limit the size of

FIG. 7.

FIG. 8.

FIG. 9.

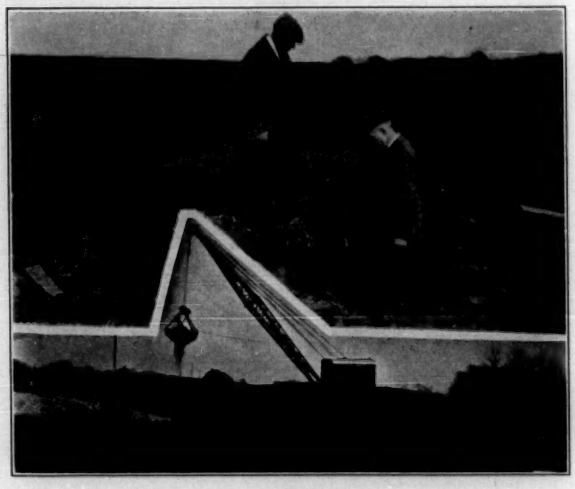


the wire to a reasonable magnitude and the actual efficiency is lowered accordingly.

Regarding the pick-up system, it will be noticed from the wiring in Fig. 1 that there exists no connection to the power coil or high potential side of the system, incidentally protecting the operator from receiving shocks through the telephone.

It is not necessary to employ more than approximately 100 turns of wire in each of the pick-up coils and the impedance of the telephone should not differ too much from the impedance of the coils for best results. The length of the telephone-connection line should be at least 20 feet.

FIG. 10.



Upper picture shows the hole left by one of the bombs as it was destroyed.

The practicability of connecting the low-impedance pick-up system to an audio-amplifier has been given some consideration. More experimental work, however, is necessary in order to perfect the arrangement to such a degree that the

added complications of the scheme may be justified by improved performance.

It is important to notice that the individual connections to the coils must not be allowed to hang loose, but must be carefully twisted together to neutralize the effect of the field and carefully secured to the frame. Notice also in Fig. 5, how the leads are brought out of the field.

FIG. 11.

Bomb uncovered in path of sea plane towing channel.

Regarding the structural design of the instrument, it must be made as rigid as possible and at the same time it should not be heavier than can be carried by two men with ease. Not a single nail or screw should be used, since this will make perfect balancing by the simple means referred to above impossible. The method of construction is quite evident from Fig. 5 and 13.

FIELD OPERATION OF THE INSTRUMENT.

The site of the new Seaplane Towing Channel is a strip of land about 20–30 feet wide and more than 2,000 feet long, located to the north of the present buildings of the Langley Memorial Aeronautical Laboratory.

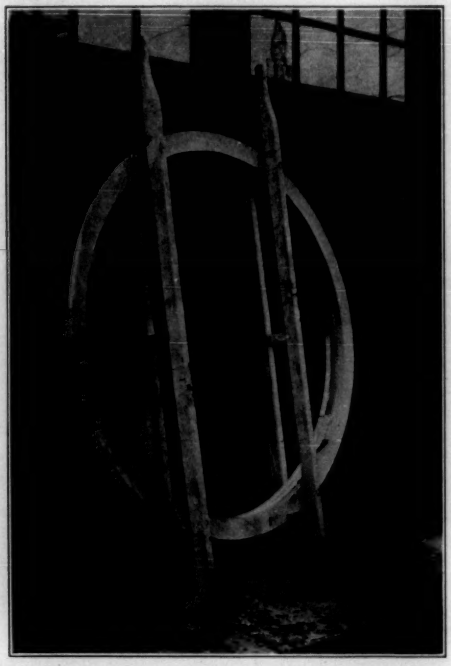
The generator was put at one end of the field and a supply line of 500-cycle current was run from this point to the instrument. Some photographs of the instrument in operation on the site are reproduced in Fig. 7 and 8.

FIG. 12.

Two men carried the Detector with the bottom coil a few inches above the ground. The operator walked behind or to one side at a distance of about 20 feet or more.

No difficulty was experienced in operation. The instrument showed no tendency to get out of balance, nor was the balancing in itself more difficult than the tuning of a single dial radio. This simplicity in operation is of considerable practical value.

FIG. 13.



Experimental power coil.

Fig. 10 is a photograph of the first bomb hole. Figure 11 shows a newly uncovered bomb, the tail and part of the body being visible. Note the peculiar method of digging employed. Figure 12 shows the explosion from a bomb being destroyed.

LANGLEY FIELD, VA.,
March 18, 1930.

THE DYNAMICAL THEORY OF ALTERNATING CURRENT MACHINERY AND THE PROBLEM OF THE STABILITY OF POWER SYSTEMS.

BY

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University of Washington.

INTRODUCTION.

The problem of the stability of systems of alternating current dynamos and motors is one of such difficulty that the behavior of such systems under disturbances in their steady state operation is still mathematically unpredictable. The problem has received little more than definition, in fact, despite its great economic importance. The purpose of the present paper is to discuss the rôle dynamical methods must of necessity play in any adequate formulation of the answer to the problem and, in particular, to give the elementary dynamical theory of the alternating current motor.

Many phenomena coming under the heading of behavior of alternating current machinery have been analyzed without reference to any basic energy function. Among them may be mentioned the phenomenon of penduling of parallel-connected alternators treated originally by Kapp, Görges and others and more generally by Hopkinson [1] and Sommerfeld,[2] and the phenomenon of unstable regulation (governing) of simple systems treated by Föppl.[3] Following the example of these early and eminent researchers, later writers apparently without exception have avoided a dynamical formulation of the theory of rotary electrical machinery. Such a formulation becomes necessary, nevertheless, as soon as the significance of certain simplifying assumptions that are always made (explicitly or implicitly) is in question. For instance, the assumption of a sinusoidal air-gap flux distribution in a motor or generator is found to apply, if that distribution is to hold

[1] *Proc. Roy. Soc.*, Vol. 72 (1903), p. 235.

[2] *E. T. Z.*, 14 (1904), p. 273, p. 291.

[3] *E. T. Z.*, 4 (1902), p. 59.

for more than one set of initial or working conditions, to an infinite momentum associated with the electrical field coördinate, i.e., to imply that the inductance of the field-winding circuit is infinite. As the amount of flux leakage in most alternating current machinery is small rather than large in comparison to the working flux, it would seem rather surprising that discrepancies between theory and experiment have never been noted were it not for the explanation that no great correlation between theory and experiment has ever been achieved in this field because of the complexity of the phenomena involved and the crudity of the methods for their measurement. More exact methods of measurement are here given which should permit, at least, a more definite reference to particular phenomena of discrepancies between theory and experiment.

The scope and generality of the dynamical method is limited only by the accuracy of the specifying functions (energy, dissipation and activity) and this accuracy only by the restriction that the specific physical constants μ and ϵ be constant. It is conceivable that such functions might describe small scale mechanism in an integrated and statistical way to take account of hysteresis as that phenomenon has its origin in the temporally discontinuous strain of iron crystals in amorphous frictionous cements.[4] These phenomena (hysteresis and variable specific physical constants) may be left for the present to be taken into account in future theoretical developments.

A typical stability problem is as follows: given, in the simplest case, an alternating-current generator and an alternating-current motor connected together in steady-state operation, will the sudden switching in of other apparatus in parallel with the motor throw it out of step and bring it eventually to a stop? An answer can be given if it be known that the steady-state rotary motion of the motor in the new system is not possible; if, on the other hand, such rotary

[4] The connection between Hook's law for crystal syntropes and their electrical properties is fundamental in the lattice theory of rigid bodies. The effects of elastic stress on magnetic properties of metals are, of course, well known, but a lattice theory has not been worked out. For our purpose, it is only necessary to imagine that the shape-strains are relieved in a discontinuous fashion characteristic of systems where static is greater than non-static friction.

motion of the motor be possible we cannot say that it will of necessity result until we have decided the question, will the final motion of the motor be one of rotation, given the initial motions, currents, etc., of the new system?

An ideal criterion of stability would be at hand if an equation were known for the hypersurface (n-spread) of critical initial values of the $n + 1$ dimensional space of n generalized coördinates describing the configuration of the system and the one time coördinate, but present mathematical theory is deficient here and we are obliged to have recourse to the following practical answer: if the rotary motion of the motor after a certain time be sufficiently close to that given by the steady-state solution of the problem and if for longer times the approximation be closer, then the persistence of rotary motion can be inferred with assurance. If it be found that rotary motion will persist no matter at what instant the switch be thrown, the new system will be said to be *stable under the switching operation in question*. The ultimate motion may depend upon the time at which the switch is thrown, in which case one may speak of *quasi-stability* under the switching operation. The new system may be said to be *completely unstable under the switching operation* when the rotary motion of the motor cannot survive the switching operation for any time of throwing the switch.

The motion of the system will be described at any instant by a set of differential equations which are obtained from certain functions of specification by the Lagrangean or Hamiltonian formulæ (or rather an extension of the same to dissipative systems), supplemented by more or less empirical formulæ describing the behavior of governors, field exciters, etc.

The steady-state motions are deduced from these equations and furnish the initial values of the variables of the set of differential equations describing the motion of the new system brought into being with the throwing of the switch. The history of the new system subsequent to that time can be traced from these initial values only by some method for the numerical solution of systems of differential equations. Such a method, somewhat similar to Runge's, has been developed recently by Prof. John Ballantine to a point where it appears

that the greater part of the labor involved could be done by computing machines of the Marchand type if a certain cascade arrangement of carriage registers could be made available and equipped for electrical operation. We have here, then, a promise of a practical method for the numerical study of projected systems of dynamos and motors and their accessories.

1. DYNAMICAL SPECIFICATION OF A SINGLE-PHASE MOTOR.

It is desired to consider first the theory of a single-phase motor connected to an infinite bus, i.e. connected to line terminals maintained at a sinusoidal potential difference e. The motor will be dynamically specified by the following functions: [5]

$$\left.\begin{aligned} \mathbf{T} &= \tfrac{1}{2}l(\eta)\dot{q}^2 + m(\eta)\dot{q}\dot{Q} + \tfrac{1}{2}L(\eta)^2\dot{Q} + \tfrac{1}{2}\lambda\dot{\eta}^2 \\ \mathbf{V} &= 0 \\ \mathbf{S} &= \tfrac{1}{2}r\dot{q}^2 + \tfrac{1}{2}R\dot{Q}^2 + \tfrac{1}{2}\rho\dot{\eta}^2 \\ \mathbf{U} &= e\dot{q} + E_0\dot{Q} - f\dot{\eta} \end{aligned}\right\} \qquad (1)$$

where $\dot{q}$ is the current in the winding connected to the mains (to be called the external circuit of the motor), r the resistance of this winding, $l(\eta)$ its self-inductance, $m(\eta)$ its inductance mutual with the field winding (or internal circuit) carrying the current $\dot{Q}$ and having resistance R and inductance $L(\eta)$, η the angular coördinate giving the position of the rotor, ρ the frictional resistance to motion and λ the moment of inertia of the rotor, E_0 the (generally constant) emf. impressed on the internal circuit and f the mechanical torque exerted by the motor on its load.

Applying the formula [6]

[5] A forced, dissipative dynamical system is specified properly by four functions: kinetic and potential energy functions $\mathbf{T}$ and $\mathbf{V}$, a Rayleighean dissipation function $\mathbf{S}$ (after *Strutt*) and an activity function $\mathbf{U}$ and possibly by auxiliary relations of catenation or leverage.

[6] Hamilton's principle of least action for a system of this type has expression in the form

$$\int_{\sigma} \mathbf{L}dt + \int_{t_0}^{t_1} \Sigma \frac{\partial \mathbf{A}}{\partial \dot{q}_i} \delta q_i dt = 0$$

where the first integral is taken over a trajectory and an adjacent path (in q-space) given by the variation having points of coincidence at times t_0 and t_1.

$$\frac{d}{dt}\left(\frac{\partial \mathbf{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathbf{L}}{\partial q_j} = \frac{\partial \mathbf{A}}{\partial \dot{q}_j}, \tag{2}$$

where $\mathbf{L} = \mathbf{T} - \mathbf{V}$ and $\mathbf{A} = \mathbf{U} - \mathbf{S}$, the net activity of external forces F_j and frictional forces on the system, we have the equations of motion of the single-phase motor:

$$\left.\begin{aligned} r\dot{q} + \frac{d}{dt}(l\dot{q} + m(\eta)\dot{Q}) &= e(t) \\ R\dot{Q} + \frac{d}{dt}(L\dot{Q} + m(\eta)\dot{q}) &= E_0 \\ \lambda\ddot{\eta} + \rho\dot{\eta} - \frac{\partial}{\partial \eta}(\tfrac{1}{2}l\dot{q}^2 + \tfrac{1}{2}L\dot{Q}^2 + m\dot{q}\dot{Q}) &= -f \end{aligned}\right\}. \tag{3}$$

It will appear later that operation of the motor is possible only when the momentum associated with the coördinate Q is maintained above a certain level and this brings with it the requirement that $\dot{Q}$ be similarly maintained. Because of, and only because of, energy dissipation due to resistance is the e.m.f. E_0 necessary. If both E_0 and R are evanescent the coördinate Q becomes ignorable. For simplicity, l and L may be assumed constant in our idealization of the situation; the behavior of such an idealized machine may be studied first and the effects arising from a dependence of l and L upon η and those due to dissipation be taken up later.

If we place R and E_0 equal to zero in (1), the Lagrangean formula for the coördinate Q gives

$$\frac{d}{dt}\left(\frac{\partial \mathbf{L}}{\partial Q}\right) = \frac{d}{dt}(m\dot{q} + L\dot{Q}) = 0, \tag{4}$$

Since the second integrand is not a complete variation there are no integral invariants of "relative" type and the equations

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$$
$$\frac{dp_i}{dt} = \frac{\partial A}{\partial q_i} - \frac{\partial H}{\partial q_i}$$
$$H = \Sigma p_i q_i - \mathbf{L}$$

corresponding to the Hamiltonian first-order system are not subject to a contact transformation. Birkhoff (Am. Journ. Math. 49 (1927), p. 1) insists that certain properties of stability of the solutions are more significant than the variation principle which gives rise to the equations of motion in the dynamics of dissipationless systems. These properties of stability are no less characteristic of dissipative systems such as the present.

whence

$$m\dot{q} + L\dot{Q} = \beta = \text{const.}, \tag{5}$$

and the equations of motion of the idealized motor become

$$\left.\begin{aligned} l\ddot{q} + r\dot{q} + \frac{d}{dt}\beta m(\eta) - \dot{q}m^2(\eta)/L &= e(t) \\ \lambda\ddot{\eta} + \rho\dot{\eta} - \dot{q}\frac{\partial}{\partial\eta}(\beta m - \dot{q}m^2/2)/L &= -f(t) \end{aligned}\right\}, \tag{6}$$

obtainable from

$$\frac{d}{dt}\left(\frac{\partial \mathbf{R}}{\partial \dot{q}_j}\right) - \frac{\partial \mathbf{R}}{\partial q_j} = \frac{\partial \mathbf{A}}{\partial \dot{q}_j}, \tag{7}$$

where the Routhean function $\mathbf{R}$ is defined as

$$\mathbf{R} = \mathbf{L} - \dot{Q}\beta = \tfrac{1}{2}l\dot{q}^2 - (\beta - m\dot{q})^2/2L. \tag{8}$$

2. THEORY OF MEASUREMENTS.

The equations (1) will specify a real motor in the absence of hysteresis, variability of μ and eddy currents. Since these effects are always present, the magnitudes of the quantities l, L, m, r, R always depend to some degree upon the conditions of operation.

Preliminary measurements of these quantities, on the assumption that they are functions of η only, may be made by removing the exciting e.m.f., E_0, rigidly clamping the rotor and applying the theory of measurements on a transformer specified by the functions

$$\left.\begin{aligned} \mathbf{T} &= \tfrac{1}{2}l\dot{q}_1^2 + m(\eta)\dot{q}_1\dot{q}_2 + \tfrac{1}{2}L\dot{q}_2^2, \\ \mathbf{V} &= 0, \\ \mathbf{S} &= \tfrac{1}{2}r\dot{q}_1^2 + r_{12}\dot{q}_1\dot{q}_2 + \tfrac{1}{2}R\dot{q}_2^2, \\ \mathbf{U} &= e\dot{q}_1. \end{aligned}\right\} \tag{9}$$

The steady-state currents under an impressed sinusoidal e.m.f. satisfy the equations

$$\left.\begin{aligned} Z_1\dot{q}_1 + Z_{12}\dot{q}_2 &= e, \\ Z_{21}\dot{q}_1 + (Z_2 + z_l)\dot{q}_2 &= 0, \end{aligned}\right\} \tag{10}$$

where e and $\dot{q}$ now are clock-diagram vectors and where [7]

$$\left.\begin{aligned} Z_1 &= r + j\tilde{\omega}l, \\ Z_{12} = Z_{21} &= r_{12} + j\tilde{\omega}m, \\ Z_2 &= R + j\tilde{\omega}L, \end{aligned}\right\} \tag{11}$$

z_l is the secondary load impedance and r_{12} a mutual dissipation term introduced to account in a rough way for core losses.

The same transformer with conventional load is also specified by the quantities

$$\left.\begin{aligned} \mathbf{2T} &= l_0\dot{q}_0^2 + l_1\dot{q}_1^2 + (l_2 + l_l)\dot{q}_2^2, \\ \mathbf{2V} &= C_l\dot{q}_2^2, \\ \mathbf{2S} &= r_0\dot{q}_0^2 + r_1\dot{q}_1^2 + (r_2 + r_l)\dot{q}_2^2, \\ \mathbf{U} &= (e - e_1)\dot{q}_1 - e_2\dot{q}_2 - e_0\dot{q}_0, \end{aligned}\right\} \tag{12}$$

together with the relations:

$$n_0\dot{q}_0 + n_1\dot{q}_1 + n_2\dot{q}_2 = 0, \tag{13}$$

$$e_0/n_0 = e_1/n_1 = e_2/n_2. \tag{14}$$

The equations (12) specify three forced, dissipative, linear oscillators without mutual coupling but under the constraint specified by (13) and external forces e and $-e_0$, $-e_1$, $-e_2$ among the latter of which exists a leverage relation (14); [8] (13) is derived from the circuital work theorem on the assumption that $\int \mathbf{H}\cdot d\mathbf{s}$ is vanishingly small for all circuits linking the three windings in question; (14) is derived on the assumption that this field of vanishingly small energy content is linked with the three windings in the turns-ratio $n_0 : n_1 : n_2$.

The formulas (2) applied to (12) give, with the use of (13) and (14):

$$\left.\begin{aligned} \left(z_1 + \left(\frac{n_1}{n_0}\right)^2 z_0\right)\dot{q}_1 + \frac{n_1 n_2}{n_0^2} z_0\dot{q}_2 &= e, \\ \frac{n_1 n_2}{n_0^2} z_0\dot{q}_1 + \left(z_2 + z_l + \left(\frac{n_2}{n_1}\right)^2 z_0\right)\dot{q}_2 &= 0, \end{aligned}\right\} \tag{15}$$

[7] Complex quantities of the type occurring in (11), where $j = \sqrt{-1}$, were introduced originally into the theory of alternating currents by Prof. A. E. Kennelly in a paper read before the American Institute of Electrical Engineers in April, 1893.

[8] The quotients in (14) are each equal to $-\lambda$ where this λ is Lagrange's multiplier employed when we regard our system as a non-holonomic one given by L and S of (12) and subject to the constraint (13).

where

$$\left.\begin{aligned} z_1 &= r_1 + j\bar{\omega}l_1, \\ z_2 &= r_2 + j\bar{\omega}l_2, \\ z_0 &= r_0 + j\bar{\omega}l_0 \end{aligned}\right\} \tag{16}$$

are Kennelly's operators and the $\dot{q}$'s and e are clock-diagram vectors, as before. Comparing (15) with (10) we have:

$$\left.\begin{aligned} r &= r_1 + \left(\frac{n_1}{n_0}\right)^2 r_0, \\ l &= l_1 + \left(\frac{n_1}{n_0}\right)^2 l_0, \\ R &= r_2 + \left(\frac{n_2}{n_0}\right)^2 r_0, \\ L &= l_2 + \left(\frac{n_2}{n_0}\right)^2 l_0, \\ r_{12} &= \frac{n_1 n_2}{{n_0}^2} r_0, \\ m &= \frac{n_1 n_2}{{n_0}^2} l_0. \end{aligned}\right\} \tag{17}$$

A method for determining the leakage impedances z_1 and z_2, the coupling ratio n_1/n_2 and the exciting impedance z_0 under any steady-state condition has been given elsewhere.[9]

Assuming, then, that these quantities have been determined for various values of η, the question arises as to how far the results may be used in the theory of the motor. The fictitious resistance r_0 introduced to take into account core losses in the transformer must be equated to zero in the case of the motor where parasitic losses will have to be left to be taken into account in some future refinements in the theory. The inductance l_0, however, is a more dynamically valid conception. The quantities l, L and m computed from the transformer measurements apply exactly when effects due to the variability of μ with H are not present. When such effects are present, the quantities l, L and m may be assumed approximately to apply when the transformer load z_l is such that the current $\dot{q}_1$, in magnitude and phase, is equal to that taken by the motor; L, however, may be measured

[9] *Phil. Mag.*, 47 (Jan., 1924), p. 241.

directly, and perhaps with more valid results, with the external circuit open but with the internal circuit carrying the current E_0/R.

3. FIELD DISTORTION.

When L and l are not functions of η and when l_2 is very large, both bracketed quantities in (6) become equal to $\dot{Q}_0 m(\eta)$, where

$$\dot{Q}_0 = \lim_{L \to \infty} \beta/L. \tag{18}$$

The limiting form of the equations (6) then is

$$\left.\begin{aligned} l\ddot{q} + r\dot{q} + \dot{Q}_0 m'(\eta)\dot{\eta} &= e, \\ \lambda\ddot{\eta} + \rho\dot{\eta} - \dot{Q}_0 m'(\eta)\dot{q} &= -f. \end{aligned}\right\} \tag{19}$$

The same limiting form pertains in the case where E_0, R are very great or when, by any other means, $\dot{Q}$ is held constant.

In a less extreme case, more complicated reactions are involved. Returning to the equations (6) we may assume that for some test current $\dot{q}^*$ and appropriate load f^*, the flux is sinusoidal:

$$(\beta m - \dot{q}^* m^2)/n_1 L = \Phi^* = \Phi_0^* \sin \nu(\bar{\omega}t - \phi), \tag{20}$$

where, as before, n_1 is the effective number of turns of the external winding linked with the flux Φ and where ν is the number of pole pairs of the external winding. The e.m.f. e may be assumed to have the form $e_0 \cos \nu\bar{\omega}t$; it is expended in part on the counter e.m.f. $-n_1 \frac{d\Phi^*}{dt}$. From equation (20) it appears that m can be a sinusoidal function of the time only when the test current is vanishingly small and that the flux will be a sinusoidal function of the time (and of η, λ large) for the test current and for no other.

When the test current is large, corresponding, say, to the rated H. P. of the motor, the Fourier components of $m(t)$ may be computed from (20).

Writing

$$\left.\begin{aligned} m(t) &= \sum a_k \sin k\theta + b_0/2 + \sum b_k \cos k\theta, \\ \theta &= \nu(\bar{\omega}t - \phi), \\ \dot{q}^* &= \dot{q}_0^* \cos \nu(\bar{\omega}t - \gamma), \end{aligned}\right\} \tag{21}$$

and assuming (20) to be solved for m, we have

$$\begin{matrix} a_k \\ b_k \end{matrix} = \frac{1}{\dot{q}_0^* \pi} \int_0^{2\pi} \frac{\beta - \sqrt{\beta^2 - B^2 \sin\theta \cos(\theta - \theta_0)}}{\cos(\theta - \theta_0)} \begin{matrix} \sin \\ \cos \end{matrix} k\theta d\theta, \quad (22)$$

where

$$\begin{aligned} \theta_0 &= \nu(\gamma - \phi), \\ B^2 &= 4n_1 \dot{q}_0^* L \Phi_0^*. \end{aligned} \quad (23)$$

In (22) the negative root is retained because $m(t)$ must approximate $m_0 \sin \nu(\bar{\omega}t - \phi)$ for large L. To evaluate the above integral, the radical in the integrand must be expanded by the binomial theorem with the jth term in the resulting series being

$$(-)^{j+1} \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2) \cdots (\frac{1}{2} - j + 1)}{1 \cdot 2 \cdot 3 \cdots j} \times \beta^{1-2j} B^{2j} \cos^{j-1}(\theta - \theta_0) \sin^j \theta \begin{matrix} \sin \\ \cos \end{matrix} k\theta. \quad (24)$$

To integrate terms of this type we may have recourse to the formula: [10]

$$2^n \cos^n \theta = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \cos(n - 2k)\theta. \quad (25)$$

The integrand will expand accordingly into terms containing the factors

$$\int_0^{2\pi} \cos(j + 1 - 2l)(\theta - \theta_0) \cos \times (j + 2 - 2l)(\theta - (\pi/2)) \begin{matrix} \sin \\ \cos \end{matrix} k\theta d\theta = \frac{1}{4} \sum_1^4 i \int_0^{2\pi} \sin\left(c_i\theta + \frac{d_i}{d_{i'}}\right) d\theta, \quad (26)$$

where

$$\left.\begin{aligned} c_1 &= k + 2j - 4l + 3, \\ c_2 &= k - 2j + 4l - 3, \\ c_3 &= k - 1, \\ c_4 &= k + 1, \end{aligned}\right\} \quad (27)$$

[10] This formula is best established by induction. It differs from Abel's well-known formula in that the termination of the series is determined by the vanishing of the coefficient $n!/k!(n-k)!$ rather than by $n - 2k$ reaching a minimum. It is believed to be new.

$$\left.\begin{aligned} d_1 &= -(j+1-21)\theta_0 - (j+2-21)\pi/2 = d_1' + k\pi/2, \\ d_2 &= \quad (j+1-21)\theta_0 + (\qquad\quad)\pi/2 = d_2' + k\pi/2, \\ d_3 &= -(\qquad\quad)\theta_0 + (\qquad\quad)\pi/2 = d_3' + k\pi/2, \\ d_4 &= \quad (\qquad\quad)\theta_0 - (\qquad\quad)\pi/2 = d_4' + k\pi/2. \end{aligned}\right\} \tag{28}$$

The d's will differ from the primed d's by $k\pi/2$, as indicated, but the c's will be independent of the selection of the sine or cosine in the integrand on the right-hand side of (26). When the c's are odd, the integrals on the right-hand side of (26) are zero; accordingly, from the relation of k to the c's given in (27), the Fourier series will contain only odd terms.

Another physically significant relation may be obtained from a consideration of the condition under which a_k and b_k are real. The maximum value of $B^2 \sin\theta \cos(\theta - \theta_0)$ must be less than β^2, i.e., we must have [11]

$$\beta^2 > B^2(1 + \sin\theta_0)/2. \tag{30}$$

These results hold with unessential differences when l and L are functions of t; changing Φ^* to $[\beta m - \dot{q}^*(m^2 - lL)]/n_1 L$ affects only the magnitudes of B and θ_0.

If a function be defined as *symmetric periodic* when its value recurs with sign reversed, after half a period, for every value of the independent variable, then, as we have just found, $m(t)$ must be a symmetric periodic function of the time when the coördinate Q is ignorable and when it is required that the current $\dot{q}$ be sinusoidal. In most motors the windings are disposed and other parts constructed with sufficient symmetry to insure that m will be a symmetric periodic function of η and, when λ is very large, of t. It may be shown that the mean value of the current in the internal circuit depends upon this symmetry.

Writing

$$q(t) + q\left(t + \frac{\tau}{2}\right) = x$$

and

$$Q(t) - Q\left(t + \frac{\tau}{2}\right) = X,$$

[11] From this equation it is possible to deduce that no induction motor can be made which will not distort the current in its external circuit. Our idealization of an induction motor is one for which $\beta = 0$.

where τ is the period, we have from (3), if $e(t)$ be symmetric periodic, the equations

$$\left.\begin{aligned} rx + l\dot{x} + m\dot{X} &= 0, \\ RX + L\dot{X} + m\dot{x} &= 0, \end{aligned}\right\} \tag{31}$$

the steady-state solution of which is $x = X = 0$. Whence $\dot{q}$ is symmetric periodic, and the period of $\dot{Q}$ is half that of $\dot{q}$.

In arriving at the equations (31) we have assumed that $L(t) = L\left(t + \frac{\tau}{2}\right)$ and that $l(t) = l\left(t + \frac{\tau}{2}\right)$; i.e., that the period of these quantities is half that of m since the relative positions of the regularly spaced cores of stator and rotor have a frequency of recurrence twice that of the windings which they carry.

The dependence of l and L on η and hence on the time may be in part an extra-dynamical phenomenon and to be accounted for in terms of the phenomena of ferro-magnetism. Those effects in the external circuit produced by the relative motion and current of the internal circuit which depend upon the polarity of the connecting magnetic flux are of the type contemplated in the Faraday-Maxwell theory; they may be completely accounted for by the term in $\dot{q}$ and $\dot{Q}$ in the kinetic energy. Those effects depending upon the absolute intensity of the field cannot be so accounted for as they result in variation in the inductance $l(\eta)$ away from the functional form that function would have if purely geometric.

Similarly, those effects in the internal circuit which depend upon the absolute intensity of the magnetic field must be of double frequency.

Since X of (31) is zero, we have that $\dot{Q}(t) = \dot{Q}\left(t + \frac{\tau}{2}\right)$ and that $\dot{Q}(t) = \dot{Q}_0 + \dot{\overset{\cdot}{Q}}$, where $\dot{\overset{\cdot}{Q}}$ is symmetrical periodic. Integrating both sides of the second of (3) with respect to t from 0 to τ, we find that the mean value $\dot{Q}_0$ of the current is E_0/R.

When λ is not very large, it becomes desirable to show that given m a symmetric periodic function of η that m will remain a symmetric periodic function of t and that the results

just found with regard to $\dot{q}$, Q, l and L hold. Writing $\eta = \tilde{\omega}t + X(t)$ the results will follow if

$$X\left(t + \frac{\tau}{2}\right) - X(t) \equiv y(t) = 0.$$

The third equation of (3) yields

$$\lambda\ddot{y} + \rho\dot{y} - \frac{\partial}{\partial\eta}(\mathsf{L}_{\tau/2} - \mathsf{L}) = f - f_{\tau/2}.$$

From this we see that the results are consistent, at least, with the solution $y = 0$, if the load reaction is such that $f\left(t + \frac{\tau}{2}\right) = f(t)$.

4. LIMITS OF STEADY-STATE OPERATION.

The limits of steady-state operation of the a. c. motor may be obtained from the loci [12] of the clock-diagram vectors representing current, magnetic flux and impressed e.m.f. when the latter are sinusoidal functions of the time for the whole range of possible steady-state operations, i.e. when $\dot{Q}$ and l are constant and when $m(t)$ is sinusoidal. Under the restrictions just mentioned, the relations between current, electro-motive force, mechanical power, etc., have simple algebraic formulation.

The average work per period being done by the external forces e and $-f$ upon the ideal motor is

$$\overline{W} = \frac{1}{\tau}\int_0^\tau e\dot{q}dt - \frac{1}{\tau}\int_0^\tau f\dot{\eta}dt = r\dot{q}_{\text{eff.}}^2 + \rho\dot{\eta}_{\text{eff.}}^2, \qquad (32)$$

obtained by multiplying the first of equation (19) by $\dot{q}$, the second by $\dot{\eta}$, adding and integrating. A further result is that

$$\frac{1}{\tau}\int_0^\tau e^*\dot{q}dt = \frac{1}{\tau}\int_0^\tau f\dot{\eta}dt + \rho\dot{\eta}_{\text{eff.}}^2, \qquad (33)$$

where

$$e^* = \frac{d}{dt}n_1\Phi^* = e_0^*\cos\nu(\tilde{\omega}t - \phi). \qquad (34)$$

[12] The first to use such graphical constructions appears to have been T. H. Blakesley, *Collected Papers*, London, 1885. Their present form is due principally to Blondel: "Theorie des moteurs synchrones," Paris, 1895.

A very general type of load will be represented by a power series in $\dot{\eta}$ in place of $f(t)$ if the inertia of the load be included in λ. Upon making the transformation of variable

$$\eta(t) = \bar{\omega}t + \xi(t), \tag{35}$$

(33) goes over into

$$\frac{I}{\tau}\int_0^\tau e^*\dot{q}dt = \frac{I}{\tau}\int_0^\tau f\xi dt + \rho\bar{\omega}^2 + \frac{\rho}{\tau}\int_0^\tau \xi^2 dt. \tag{36}$$

In most motors the moment of inertia λ is so large that the maximum value of the variation ξ in the angular velocity $\dot{\eta}$ is small in comparison to the mean angular velocity $\bar{\omega}$. The first and last terms of (36), accordingly, may be neglected unless the coefficients in the power series for $f(t)$ are large in comparison to ρ which may be assumed not to be the case. Evaluating the left-hand side of (36), one obtains $\frac{1}{2}e_0^*\dot{q}_0 \cos \nu(\gamma - \phi)$ so that, for a general load, one may write

$$e_0^*\dot{q}_0 \cos \nu(\gamma - \phi) = 2p_l. \tag{37}$$

The power expended upon the load, p_l, is equal to $\rho\bar{\omega}^2$ when the load is the ideal type.

Equation (37) together with the equation

$$e - e^* = r\dot{q} + l\ddot{q}, \tag{38}$$

gives the important results

$$e_0^{*2}r + p_l z^2 - e_0 e_0^*(r \cos \nu\phi + x \sin \nu\phi) = 0, \tag{39}$$

$$\tan \nu\gamma = \frac{p_l x - e_0^{*2} \sin \nu\phi \cos \nu\phi}{p_l r + e_0^{*2} \sin^2 \nu\phi}, \tag{40}$$

$$\dot{q}_0^2 r - \dot{q}_0 e_0 \cos \nu\phi + p_l = 0, \tag{41}$$

$$e_0^2 - 2e_0 e_0^* \cos \nu\phi + e_0^{*2} = \dot{q}_0^2 z^2, \tag{42}$$

where

$$\left.\begin{aligned} x &= \nu\bar{\omega}l, \\ z^2 &= r^2 + x^2, \\ e &= e_0 \cos \nu\bar{\omega}t, \\ e^* &= e_0^* \cos \nu(\bar{\omega}t - \phi), \\ \dot{q} &= \dot{q}_0 \cos \nu(\bar{\omega}t - \gamma). \end{aligned}\right\} \tag{42a}$$

Equations (41) and (42) were obtained first, apparently, by J. Hopkinson, and are well known. The derivation of (40)

and (41) may be indicated: all the cosines in (37) and (38) are expanded and in (38) the terms in sin $\nu\tilde{\omega}t$ and cos $\nu\tilde{\omega}t$ on one side of the equation are equated, respectively, to the corresponding terms on the other. It is then possible to solve for sin $\nu\gamma$ and cos $\nu\gamma$ using one of these equations and the expanded form of (37). The equation (41) is obtained with little more difficulty and, to save space, may be left to the reader to verify.

For a given motor at given excitation, the angle ϕ is given by (39). The equation reveals the well-known result due to Blondel that the polar locus of the point $(e^*, \nu\phi)$, with vector e_0 as pole, is a circle with center at $(\frac{1}{2}e_0 \sec \nu\phi, \arctan x/r)$ and radius equal to $\frac{1}{2}\sqrt{e_0^2 - 4rp_1} \sec \nu\phi$; the angle $\nu\phi = \arctan x/r$.

The value of ϕ obtained from (39) may be substituted in (40) to give γ. This equation shows that the current will be leading or lagging according as the numerator is positive or negative and seems to be new.

Equation (41) gives $\dot{q}_0$, γ known. It shows the well-known result that the polar locus of $\dot{q}_0$ is also a circle with center at $(e_0/r, 0)$ and radius equal to $\sqrt{e_0^2 - 4p_1r}/2r$.

The maximum and minimum possible values of e_0^* are obtained easily from (39) and are

$$(e_0 \pm \sqrt{e_0^2 - 4rp_1})z/2r,$$

the plus sign giving the upper limit, the negative sign the lower. For the larger value, γ is given by

$$\tan \nu\gamma = -\frac{2rxe_1}{r^2(e_0 - e_1) + x^2(e_0 + e_1)}, \tag{43}$$

where

$$e_1 = \sqrt{e_0^2 - 4rp_1}. \tag{44}$$

In this case the current is leading and the motor is operating as a synchronous condenser. For the smaller value, γ is given by

$$\tan \nu\gamma = \frac{2rxe_1}{r^2(e_0 + e_1) + x^2(e_0 - e_1)}. \tag{45}$$

In this case the current is lagging since tan $\nu\gamma$, and therefore γ is positive.

In both cases equivalent circuit elements are easily computed. When the motor is running as a synchronous condenser, an equivalent circuit (equivalent as far as the magnitude and phase of the current is concerned) will be a resistance in series with a condenser. For such a circuit, assuming current and voltage to have the form (42a)

$$\tan \nu\gamma = -1/\nu\tilde{\omega}CR,$$
$$\dot{q}_0 = e_0/\sqrt{R^2 + (\nu\tilde{\omega}C)^{-2}},$$

where R is the resistance of the equivalent series circuit and the C the capacity. Equating with the right-hand side of (40) and with the value of $\dot{q}_0$ obtained from (42), respectively, we have

$$R = 2re_0 \frac{r^2(e_0 - e_1) + x^2(e_0 + e_1)}{r^2(e_0 - e_1)^2 + x^2(e_0 + e_1)^2},$$

$$\nu\tilde{\omega}C = \frac{r^2(e_0 - e_1)^2 + x^2(e_0 + e_1)^2}{4r^2xe_0e_1}.$$

This last equation shows that when either ρ or r is small that a large capacity effect may be had by making x large in comparison to r since then e_1 approximates e_0 and the right-hand side approaches x/r^2, or C approximates $x/\nu\tilde{\omega}r^2$.

When the motor is running with e_0^* at its lower limit there is an equivalent circuit consisting of a resistance in series with an inductance L. For such a circuit $\tan \nu\gamma = \nu\tilde{\omega}L/R$ and $q_0^2 = e_0^2/(R^2 + (\nu\tilde{\omega}L)^2)$ and one obtains

$$R = 2re_0 \frac{r^2(e_0 + e_1) + x^2(e_0 - e_1)}{r^2(e_0 + e_1)^2 + x^2(e_0 - e_1)^2},$$

$$\nu\tilde{\omega}L = \frac{4r^2xe_0e_1}{r^2(e_0 + e_1)^2 + x^2(e_0 - e_1)^2}.$$

5. $\dot{Q}$ AND 1 VARIABLE; CONCLUSION.

The equations (39), (40), (41) and (42) were obtained on the assumption that l and $\dot{Q}$ are constant; when not constant, the limits of steady-state operation become narrower and more difficult to determine and have never received adequate treatment. If $\dot{Q}$ be held constant and l be variable, say of the form

$$l_0 + l_1 \sin\left(2\nu(\tilde{\omega}t - \phi) - \frac{\pi}{2}\right)$$

(in steady-state operation), then the current will be distorted

away from a pure sine function of the time and the amplitude of the ground oscillation of the Fourier spectrum will be decreased and, since only this ground oscillation can contribute to the right-hand side of (33), the steady-state range of operation of the motor will be reduced. Moreover, the motor will get out of step easier under transient oscillations. When $\dot{Q}$ is also variable, that is, when the equations (3) rather than (19) apply, the stability of the motor is further reduced because of the energy drain due to the term $\frac{1}{2}R(\dot{Q}_0 + \dot{\bar{Q}})^2$ in the dissipation function. For, since $\dot{Q}$ is symmetric periodic, the average work per period done upon the system by the force E_0 is given by

$$\frac{1}{\tau}\int_0^\tau E_0\dot{Q}dt = \frac{1}{\tau}\int_0^\tau E_0\dot{Q}_0 dt.$$

Multiplying both sides of the second equation of (3) by $\dot{Q}_0$ and integrating from 0 to τ and dividing by τ, one obtains RQ_0^2.

The energy drain represented by the term $\frac{1}{2}R\dot{Q}_0^2$ in the dissipation function, then, is equilibrated by the energy fed into the system represented by the term $E_0\dot{Q}$ in the activity function. The energy drain represented by the terms $R\dot{Q}_0\dot{\bar{Q}} + \frac{1}{2}R\dot{\bar{Q}}^2$ in the dissipation function must be equilibrated by the only remaining positive source of energy, namely, the impressed electro-motive force e.

These considerations have practical implication, but before definite recommendation for the improvement of the stability of power transmission systems can be ventured, certain more difficult dynamical questions have to be answered. Among them may be mentioned the question of the *adaptability* of a motor or synchronous condenser, i.e., the question as to how long a motor will take to adjust itself to new steady-state working conditions and in what way the characteristics of the internal circuit determine this adaptability. It is clear that, if means are provided to keep $\dot{Q}$ absolutely constant, the function of damping windings will be thwarted. The nature of the compromises necessary in the best possible machine, the statistical and economic relationships between transient and steady-state stability and connected questions are beyond the scope of the present paper.

Einstein on Space. (*Science*, June 13, 1930, p. 608 and *Nature*, June 14, 1930, p. 897.) On June 6 Einstein lectured in German at University College, Nottingham. An interpreter translated his address into English section by section. At the conclusion of the lecture announcement was made and enthusiastically cheered that the blackboard used by the speaker and signed by him was to be varnished and preserved as a historic memento.

Einstein holds the primary concept of physics to be that of a rigid body. The idea of space was not known by the Greeks. Behind the geometry of Euclid with its displacement of triangles lay the thought of rigid bodies. With Descartes' analytical geometry science conceived the conception of a space continuum, but it was an entirely mathematical notion. Newton, dealing with force and acceleration, conferred on space a definite physical reality. A further change in the content of the space concept was introduced by Faraday and Maxwell when they discussed the electromagnetic field. This idea joined the former trio of realities, bodies, space and time, and seemed to require the setting up of the ether to account for its actions. H. A. Lorentz made great use of the ether though he robbed it of its mechanical properties. Next came the special theory of relativity. "Classical physics used three coördinates in space and one in time. It was now found necessary to unite these into the four coördinates of space-time, and give up the belief that events could be divided into categories of 'before,' 'simultaneous,' and 'after.' This prepared the way for the general theory of relativity, which dealt with the phenomena of inertia and derived the laws of motion from the geometrical structure of space, or rather of space-time, thus uniting geometry and physics in a new intimacy. The experimental verification of this theory is well known. We have now come to the conclusion that space is the primary thing and matter only secondary; we may say that space, in revenge for its former inferior position, is now eating up matter." In his endeavor to deduce all physical phenomena from the properties of space Einstein recognizes that he has not won the adherence of all physicists. "In fact, he added with a smile, they think that I am crazy on this subject." He continues, however, to seek a series of equations that will comprehend all the field of physics.

G. F. S.

DETERMINATION OF SILVER ION WITH VARIOUS CONCENTRATIONS OF HYDROGEN CHLORIDE AS PRECIPITANT.

BY

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INTRODUCTION.

Adsorption of silver ion by insoluble compounds of the metal, with the production of complex ions of the type Ag_2Cl+ (a result of the property that characterizes all silver precipitates, tendency to adsorb its own ions) is of great importance in the consideration of possible errors occurring in quantitative precipitations.[1] In the case of the halides of this univalent ion, the precipitating anion is adsorbed under certain conditions.

While little has been accomplished in the way of determining the exact structure of these so-called "solid solutions," the crystalline compound, $AgNO_3 - AgCl$ has been isolated.[2] The author has directed attention in previous work [3, 4] to the disastrous effects of occlusion upon the quantitative estimation of some compounds, and made suggestions as to how this source of inaccuracy may be removed. Of far greater significance, however, is the phenomenon of adsorption, so frequently encountered in analytical procedure. Kolthoff [5] is authority for the statement that very fine precipitates adsorb more strongly than crystalline ones. Barium sulfate is an exception, showing a pronounced inclination to adsorb foreign ions during its precipitation.

The observation that an excess of barium chloride in the

[1] Kolthoff and Furman, "Vol. Analysis," Vol. 1, pp. 160–3 (1928).
[2] Hellwig, *Z. anorg. Chem.*, **25**, 183 (1900).
[3] Germuth, *American J. Pharm.*, **99**, No. 5 (1927).
[4] Germuth, *Chemist-Analyst*, **17**, 3–7 (1928).
[5] Kolthoff and Furman, *supra vide*, p. 157.

precipitation of SO_4 — ion increases the quantity of anion adsorbed as the valence becomes higher, has been made.[6] This fact and the equivalence rule of Schulze, as well as many later theories appertaining to the different aspects of the subject, are exhaustively treated in the excellent volume on colloidal chemistry by Freundlich.[7]

In the paper here presented, the author had as his objective the determination of the effect of adding varying concentrations of hydrogen chloride precipitant to solutions containing silver ions.

PURITY OF MATERIALS.

The silver nitrate used in the experimental solutions was prepared by the action of nitric acid, made of highest grade materials in this laboratory, upon metallic silver of the finest grade obtainable. While the impurities present were most minute in quantity, corrections were made for these in the subsequent experimental portion of this work.

The hydrogen chloride was produced by distillation of the c.p. article, and solution in distilled water. Previous to employment, the HCl content of each sample of reagent was carefully estimated by titration with an alcoholic solution of 0.1 N KOH.

In a saturated solution of AgCl, $[Ag+] = [Cl-] = \sqrt{S}$, and $p_{Ag} = \frac{1}{2}p_S$, S_{AgCl} being 1.1×10^{-10}.

Potentiometric determination of the equivalence-point, employing the silver electrode, was made. The change in potential is most marked at the equivalence-point, and the titration error, it is believed, is practically negligible. Every precaution was taken to attain the highest degree of accuracy in the analysis of the experimental solutions under examination and proper correction made for temperature changes, etc.

METHOD.

The method applied was a modification of that proposed by Hillebrand and Lundell,[8] carried out in the following manner: 0.1 gm. of silver (or an equivalent of a silver salt)

[6] Weiser and Sherrick, *J. Phys. Chem.*, **23**, 205 (1919).

[7] Freundlich, "Kapillar Chemie," 2d Ed. (1922).

[8] Hillebrand and Lundell, "Applied Inorganic Analysis," 1929.

is dissolved in 200 ml. of 1:99 HNO_3; the proportion of Ag to HNO_3 is maintained though different quantities of the metal or its salts are examined. Interfering substances must be absent. The mixture is heated to 60–62° C., and HCl reagent added *slowly* with *constant stirring*. The contents of beaker are kept warm until the precipitate settles, and then allowed to stand in the dark for 12 hours. The supernatant liquor is poured through a weighed Gooch crucible, and the precipitate washed by decantation with 0.01 N HCl, and transferred to the crucible. The crucible and contents are heated to 100° C. for 3 hours, and then at 148–150° C., to constant weight. It is cooled in a desiccator, and weighed as AgCl.

The silver chloride remaining in solution was calculated from solubility data [9, 10] while that present on the walls of the container or dissolved during the process of washing was determined by use of the nephelometer with appropriate standards.[11]

EXPERIMENTAL.

Experimental solutions of the purified silver nitrate (the silver content of which was most carefully determined) were prepared in series of ten, each sample containing a larger amount of silver than the preceding. The normality of the hydrogen chloride reagent was strictly regulated by meticulous examination; the concentration of the acid was changed for each series of tests. Table I shows the results obtained by the employment of 0.1 N HCl as precipitant in various solutions of the soluble silver salt.

Further experimentation developed the fact that 0.15 N HCl produces results probably slightly more accurate than when the more dilute reagent is employed. Incidentally, as might be expected, the solubility of the silver chloride is much higher, also, when 0.1 N HCl is used as precipitating agent, thus increasing the proportion of the salt estimated by the nephelometer.

[9] Whitby, *Z. anorg. Chem.*, **67**, 108 (1910).
[10] Richards and Wells, Carnegie Inst. of Wash. Pub., No. 28.
[11] Yoe, "Photometric Chemical Analysis," Vol. II (1929).

TABLE I.

Determination of Silver Ion with Hydrogen Chloride.

Concn. of reagent—0.101 N HCl.

	Amount of Ag Present (Gm.).	Amount of Ag Found (Gm.).	Difference.
Sample 1	0.01518	0.01516	− .00002
Sample 2	0.03009	0.03008	− .00001
Sample 3	0.04511	0.04505	− .00006
Sample 4	0.06007	0.06001	− .00006
Sample 5	0.07520	0.07519	− .00001
Sample 6	0.09018	0.09019	+ .00001
Sample 7	0.10509	0.10509	.00000
Sample 8	0.12020	0.12022	+ .00002
Sample 9	0.13507	0.13502	− .00005
Sample 10	0.15010	0.15006	− .00004
Average Difference			−0.00002 gm.

TABLE II.

Determination of Silver Ion with Hydrogen Chloride.

Concn. of reagent—0.150 N HCl.

	Amount of Ag Present (Gm.).	Amount of Ag Found (Gm.).	Difference.
Sample 1	0.01516	0.01514	− .00002
Sample 2	0.03012	0.03013	+ .00001
Sample 3	0.04508	0.04510	+ .00002
Sample 4	0.06004	0.06004	.00000
Sample 5	0.07509	0.07509	.00000
Sample 6	0.09011	0.09012	+ .00001
Sample 7	0.10503	0.10505	+ .00002
Sample 8	0.12009	0.12009	.00000
Sample 9	0.13510	0.13511	+ .00001
Sample 10	0.15007	0.15007	.00000
Average Difference			+0.000005

The quantity of ionized silver remaining after the addition of the precipitant was much less than that encountered with those solutions considered in Table I. Undoubtedly, any solution of the reagent more dilute than 0.15 N exerts a solvent action on the precipitate.

Continued experimentation revealed that adsorption of Ag+ ion is increased when solutions of higher normality than those previously utilized are applied. Such effect is discernible in the data furnished in Table III.

TABLE III.

Determination of Silver Ion with Hydrogen Chloride.

Concn. of reagent—0.201 N HCl.

	Amount of Ag Present (Gm.).	Amount of Ag Found (Gm.).	Difference.
Sample 1	0.01519	0.01522	+ .00003
Sample 2	0.03008	0.03012	+ .00004
Sample 3	0.04511	0.04514	+ .00003
Sample 4	0.06009	0.06014	+ .00005
Sample 5	0.07504	0.07510	+ .00006
Sample 6	0.09007	0.09011	+ .00004
Sample 7	0.10509	0.10515	+ .00006
Sample 8	0.12011	0.12018	+ .00007
Sample 9	0.13510	0.13516	+ .00006
Sample 10	0.15008	0.15015	+ .00007
Average Difference			+0.00005

It was deemed desirable to ascertain the effect of adding a slightly more concentrated solution of hydrogen chloride as reagent in the next series of determinations. As was anticipated, adsorption here was even more marked.

TABLE IV.

Determination of Silver Ion with Hydrogen Chloride.

Concn. of reagent—0.252 N HCl.

	Amount of Ag Present (Gm.).	Amount of Ag Found (Gm.).	Difference.
Sample 1	0.01508	0.01518	+.00010
Sample 2	0.03008	0.03019	+.00011
Sample 3	0.04510	0.04519	+.00009
Sample 4	0.06005	0.06017	+.00012
Sample 5	0.07511	0.07522	+.00011
Sample 6	0.09009	0.09021	+.00012
Sample 7	0.10516	0.10528	+.00012
Sample 8	0.12008	0.12026	+.00018
Sample 9	0.13503	0.13514	+.00011
Sample 10	0.15009	0.15026	+.00017
Average Difference			+.00012

Further examination of these precipitates indicated with certainty the adsorption of Cl− ion with a smaller proportion of Ag+ ion. The amount of dissolved salt remaining after precipitation was extremely small.

In the endeavor to observe the effect of further concentration of the reagent upon the insoluble silver compound, a solution of HCl, approximately 0.3 Normal, was next employed. Contamination of these precipitates was most marked, as will be observed by a perusal of Table V.

TABLE V.

Determination of Silver Ion with Hydrogen Chloride.

Concn. of reagent—0.301 N HCl.

	Amount of Ag Present (Gm.).	Amount of Ag Found (Gm.).	Difference.
Sample 1	0.01509	0.01520	+.00011
Sample 2	0.03007	0.03022	+.00015
Sample 3	0.04509	0.04520	+.00011
Sample 4	0.06011	0.06029	+.00018
Sample 5	0.07508	0.07531	+.00023
Sample 6	0.09013	0.09038	+.00025
Sample 7	0.10502	0.10537	+.00035
Sample 8	0.12012	0.12041	+.00029
Sample 9	0.13511	0.13536	+.00025
Sample 10	0.15009	0.15033	+.00024
Average Difference			+.00022

Adsorption of Cl− ion (smaller portions of Ag+ ion were present) was readily discerned in this series of precipitates.

CONCLUSIONS.

It is believed that the employment of 0.15 N HCl, when utilized as precipitant in the determination of silver ions a silver chloride, insures results that are of a much higher degree of accuracy than when more concentrated solutions are employed. While there are but slight differences existing between the results obtained with 0.10 N HCl and 0.15 N HCl, there is a tendency for the silver atom to remain in the ionic state in the former case; this is overcome, apparently, when a reagent containing slightly higher amounts of hydrogen chloride is used. One experiences a most pronounced difference when solutions > 0.2 normality are the precipitating agents.

The results so obtained are much higher than those demanded by theoretical quantitative consideration.

SUMMARY.

It has been shown that 0.15 N hydrogen chloride solution is most suitable for employment as precipitant in the quantitative estimation of silver ion, this procedure reducing adsorption to a minimum.

When a more dilute solution of reagent is applied, there is a tendency for a portion of the silver to remain dissolved.

When hydrogen chloride possessing a greater concentration than 0.15 N is utilized in this determination, adsorption of $Cl-$ ion, and, to a lesser extent, of $Ag+$ ion, readily occurs.

The data presented here have been obtained by careful application of a recognized method, during the course of which temperature effect, and solubility data were given due consideration. Meticulous nephelometric examination was subsequently accorded each experimental sample prior to the calculation of results.

Expansion of Beams of Light during their Passage through Various Media. J. PLOTNIKOW AND L. SPLAIT. (*Phys. Zeit.*, April 15, 1930.) It is known that a transversal diffusion of light manifests itself in manners dependent on the wave-length and the nature of the medium. With media having molecules of normal size X-rays produce the Compton effect, while for the visible and ultra-violet rays the Raman effect takes place. With fine suspensions the Tyndall effect occurs. In the Raman and the Tyndall effects the light scattered sidewise is polarized. Plotnikow a year or two ago found that a beam of low frequency waves spreads out as it passes through a medium made up of molecules of considerable weight. In continuation of this work the present investigation was undertaken. Part of the radiation from a 50 ampere, D.C. arc lamp passed through a tube, then in turn through a filter, a rock salt lens and a small aperture in a screen. After this it continued through the substance under examination and fell on a photographic plate. Between the substance and the plate was interposed an opaque screen of the exact size to intercept the entire beam provided that it did not expand laterally. If the beam traverses air or water merely no light reaches the photographic plate, but distinct effects are recorded with other media in the path. Only a few seconds are needed to fix the effect on the plate. Filters were inserted in the path of the beam that permitted the passage chiefly of red with infra-red, of green and of ultra-violet radiation. Various kinds of media were used such as collodium, benzol, paraffin, paraffin oil, artificial resin, gelatine and aqueous solutions of alcohol and sugar. The diameters of the halos surrounding the central shadows of the screen intercepting the unenlarged beam varied greatly. With castor oil in green light it is about 7 cm., the diameter of the shadow being in all cases .9 cm., but for methyl alcohol in red and infra-red radiation it measures only about 2.2 cm. In the case of artificial resin the halo seems to be made up of rays, but the light surrounding the dark center in the instance of castor oil has no structure of this kind. Sometimes a dark ring is seen in the halo of light. Acetone displays a halo in infra-red and ultra-violet light but none in green. Solutions of ethyl alcohol in water ranging from 1 to 96 per cent. all showed marked halos. Even one drop in 200 cu. cm. water with the filters for infra-red and ultra-violet gave a large halo with rays. With green light there was almost no effect. No polarization of the light forming the halos was discovered.

G. F. S.

DYNAMICAL ANALYSIS OF MACHINES.*

BY

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METHOD OF MOMENTS.

In the analysis of a linkage assemblage, considerable simplification in the moment equations is possible by a judicious selection of moment centers, wherein the moments of certain components of the reactions at the joints disappear. In general, the virtual centers of the mechanism are to be selected as the moment centers and give the greatest elimination of the component reactions at the joints.

Consider the linkage shown on Fig. 62. We have three members AD, HE and BE, jointed at E and C. The virtual center of HE relative to AB is at I, the intersection of BE extended and AE. The components at C consist of R and P.

For the link HE, taking moments about I, we eliminate the thrust S along BE and the component R of the reaction of AD on HE. Therefore,

$$F \cdot HC \sin \phi = P \cdot CI, \qquad \therefore \quad F = \frac{P \cdot CI}{H \sin \phi},$$

and for the member AD, taking moments about A, we eliminate the reaction at A and the component R at C, so that:

$$W \cdot AD = P \cdot AC \qquad \therefore \quad P = \frac{W \cdot AD}{AC}.$$

Therefore,

$$F = \frac{AD}{AC} \cdot \frac{CI}{HC} \cdot \frac{W}{\sin \phi}.$$

* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

Continued from p. 240, vol. 210, August, 1930.

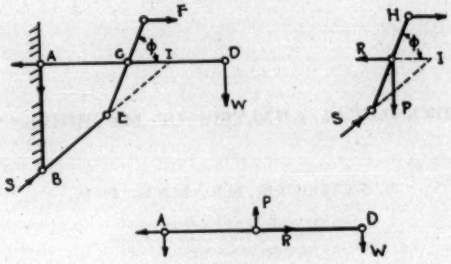


Fig. 62

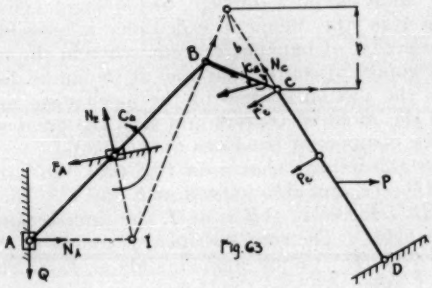


Fig. 63

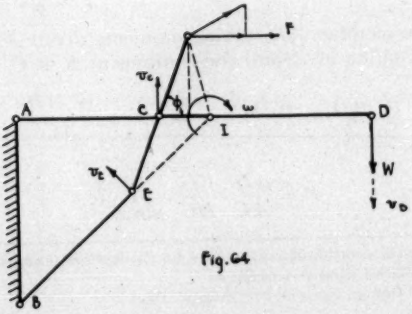


Fig. 64

An alternative attack is a consideration of only the external forces on the linkages, the reactions R and S at A and B being eliminated by taking moments about I. To determine the vertical component Q at A, we consider the forces on member AD alone and obtain $Q = \frac{CD}{AC} \cdot W$.

Introducing this value for Q in the external moments about the instantaneous center I, then:

$$F \cdot HC \sin \phi + W\left(DI - \frac{CD}{AC} \cdot AI\right) = O,$$

$$\therefore \quad F = \frac{AD\,CI}{AC\,HC} \frac{W}{\sin \phi}.$$

The advantage of the method of moments about virtual centers for a more complex mechanism can be shown by considering the Cardan Linkage Mechanism. To maintain a complete generality we will introduce the kinetic reactions of the members. These will be divided into the translatory component at the c. of g. and an inertia couple to account for the angular motion.

The Cardan Linkage, Fig. 63, consists of members AB, BC and CD. The point A of link AB slides along a constrained path, while its center of gravity E is constrained to slide along an inclined path. The virtual center of BC is at O and of AB at I, where I is the intersection of the normals to the paths of A and E. We will decompose the reactions at C and D for member CD into parallel components and a thrust N_c. The reactions at B will be decomposed into a component along $IB(N_B)$ and one perpendicular to $IB(T_B)$. The reactions of the guides at A and E are normal to their paths.

The kinetic reactions on the members are:

$F_A = -m_a \ddot{s}_E$, $\quad C_A = -I_a \ddot{\theta}$ for member AB,

$F_B = -m_b \ddot{s}_b$, $\quad C_B = -I_b \ddot{\phi}$ for member BC,

$F_c = -m_c \ddot{s}_c$ acting at the center of percussion of CD.

If now we take moments about the virtual centers O and I, we eliminate the reactions N_c, N_B and the reaction of the

guides at A and E. We also note:

$$P' = \frac{DK}{DC}P \quad \text{and} \quad F_e' = \frac{DL}{DC}F_e,$$

where DL is the distance to the center of percussion from D. Then for the moments about O:

$$P' \cdot p - F_e' \cdot OC + C_B - F_B \cdot b - T_B \cdot OB = 0$$

and for the moments about I:

$$Q \cdot AI + F_a \cdot IE + C_a - T_B \cdot IB = 0.$$

Combining and eliminating the components T_B and P', we have P in terms of Q and the kinetic reaction of the members:

$$P = \frac{DC}{p \cdot DK}\left(F_e \frac{DL}{DC} OC - C_B + F_B \cdot b + \frac{OB}{IB}(Q \cdot AI + F_a \cdot IE + C_a) \right).$$

METHOD OF VIRTUAL VELOCITIES.

In the method of Virtual Work, the reactions of the constraints are eliminated, since on the whole they do no work, so that the virtual work of the applied forces balances the virtual work of the kinetic reactions. That is, from D'Alembert's equation:

$$\Sigma[(m\ddot{x} - X)\delta x + (m\ddot{y} - Y)\delta y] = 0,$$
$$\therefore \quad \Sigma m\ddot{x}\delta x + \Sigma m\ddot{y}\delta y = \Sigma(X\delta x + Y\delta y).$$

In terms of the kinetic reactions of the members, since:

$$\Sigma m\ddot{x}\delta x + \Sigma m\ddot{y}\delta y = \Sigma m\ddot{s}\delta s + \Sigma I\ddot{\theta}\delta\theta,$$
$$\Sigma(X\delta x + Y\delta y) = \Sigma P\delta s + \Sigma L\delta\theta$$

we have:

$$\Sigma m\ddot{s} \cdot \delta s + \Sigma I\ddot{\theta}\delta\theta = \Sigma P\delta s + \Sigma L\delta\theta,$$

where the right side includes the work of extraneous forces and couples as well as internal elastic reactions and friction forces, but does not include the reactions of constraints and the mutual reactions between the parts, such as occur at the

joints between members, etc., which have a common displacement and on the whole do no work.

The constraint reactions may be viewed from the following aspect as well. If we consider the actual inertia loadings of the members reduced to equivalent loadings at the joints, then we have a system of mass points conditioned by invariable connections. If the loadings are designated by $\overline{F}_A$, $\overline{F}_B$, etc., the applied forces by P_a, P_b, etc., while the reactions due to the invariable connections; that is, the axial thrusts or tensions are designated by S_{ab}, S_{bc}, etc., and the displacement at A, B, etc., are δa, δb, etc. Then:

$$\overline{F}_A\delta a + \overline{F}_B\delta\overline{b}\cdot \cdots + \overline{P}_A\overline{\delta a} + \overline{P}_B\cdot\overline{\delta b}\cdots + \overline{S}_{ab}\cdot\overline{\delta a} - \overline{S}_{ab}\cdot\overline{\delta b}\cdots = 0$$

but $\overline{S}_{ab}(\overline{\delta a} - \overline{\delta b}) = 0$, since the difference of the projections are nil, due to the invariable connection. So that:

$$\overline{F}_A\cdot\overline{\delta a} + \overline{F}_b\cdot\overline{\delta b}\cdots + \overline{P}_a\overline{\delta a} + \overline{P}_b\cdot\delta b\cdots = 0$$

or analytically:

$$\Sigma((F_{nx} + P_{nx})\delta x_n + (F_{ny} + P_{ny})\delta y_n) = 0$$

with the condition equations:

$$f_{mn} = \sqrt{(y_m - y_n)^2 + (x_m - x_n)^2} - l_{mn} = 0,$$
$$f_{nq} = \sqrt{(y_n - y_q)^2 + (x_n - x_q)^2} - l_{nq} = 0.$$
$$\cdots \text{etc.}$$

Therefore,

$$df_{mn} = \frac{\partial f_{mn}}{\partial x_n}\cdot\delta x_m + \frac{\partial f_{mn}}{\partial y_m}\delta y_m + \frac{\partial f_{mn}}{\partial x_n}\cdot\delta x_n + \frac{\partial f_{mn}}{\partial y_n}\delta y_n = 0,$$

$$df_{nq} = \frac{\partial f_{nq}}{\partial x_n}\cdot\delta x_n + \frac{\partial f_{nq}}{\partial y_n}\delta y_n + \frac{\partial f_{nq}}{\partial x_q}\cdot\delta x_q + \frac{\partial f_{nq}}{\partial y_q}\delta y_q = 0$$
$$\cdots \text{etc.}$$

Multiplying by the multipliers, λ_{mn}, λ_{nq}, etc., and adding to the analytical expression the variations become arbitrary,

so that the coefficients must be equated to zero; that is,

$$\Sigma(F_{nx} + P_{nx}) + \lambda_{mn}\frac{\partial f_{mn}}{\partial x_n} + \lambda_{nq}\cdot\frac{\partial f_{nq}}{\partial x_n} = 0,$$

$$\Sigma(F_{ny} + P_{ny}) + \lambda_{mn}\frac{\partial f_{mn}}{\partial y_n} + \lambda_{nq}\frac{\partial f_{nq}}{\partial y_n} = 0.$$

It is evident that the constraint reactions are of the nature of the axial thrust or tension components, that is:

$$(S_{mn})_x = \lambda_{mn}\frac{\partial f_{mn}}{\partial x_n} \qquad (S_{mn})_y = \lambda_{mn}\cdot\frac{\partial f_{mn}}{\partial y_n},$$

$$(S_{nq})_x = \lambda_{nq}\frac{\partial f_{nq}}{\partial x_n} \qquad (S_{nq})_y = \lambda_{nq}\frac{\partial f_{nq}}{\partial y_n}.$$

Thus by including with the reduced inertia and applied forces at the joints the reaction of constraints between the joints, the vector sum of the reactions at any joint is nil.

In a mechanism, with one degree of freedom, the virtual displacements are obviously proportional to their actual velocities obtained from the kinematic analysis, so that we may write:

$$\Sigma\overline{P}\overline{v} + \Sigma L\omega - \Sigma\overline{m}\overline{s}\overline{v} - \Sigma I\ddot{\theta}\cdot\omega = 0$$

or

$$\Sigma\overline{P}\overline{v} + \Sigma L\cdot\omega + \Sigma\overline{F}_n\overline{v}_n = 0.$$

As an example of the method of virtual velocities, consider the previous linkage of Fig. 62.

Evidently, Fig. 64,

$$\overline{V}_f = \omega\cdot HC\sin\phi \quad \text{and} \quad V_c = \omega\cdot IC,$$

$$\therefore \quad \overline{V}_D = \frac{AD}{Ac}\cdot\omega\cdot IC.$$

Therefore,

$$W\frac{AD}{AC}\cdot\omega\cdot IC = F\cdot\omega\cdot HC\cdot\sin\phi,$$

$$F = \frac{AD}{AC}\cdot\frac{CI}{HC}\frac{W}{\sin\phi}.$$

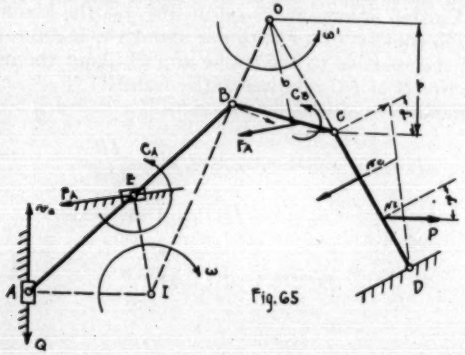


Fig. 65

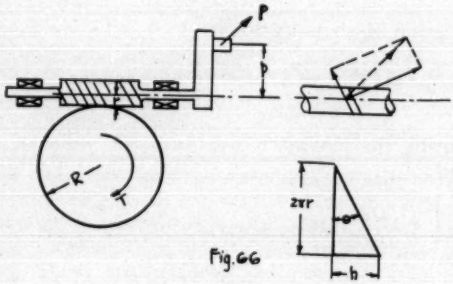


Fig. 66

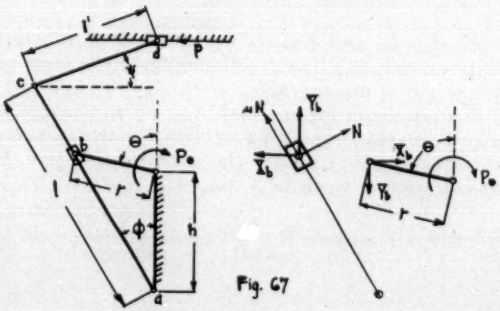


Fig. 67

In the Cardan linkage mechanism, Fig. 65, the kinematics of the system consists of a rotation ω about the instantaneous center I of AB relative to the frame and ω' about the instantaneous center O of BC relative to the frame.

The kinematic relations are, therefore,

$$\omega IB = \omega' \cdot OB = v_B, \qquad \omega' = \frac{IB}{OB}\omega,$$

$$v_A = \omega \cdot IA, \qquad v_E = \omega \cdot IE \quad \text{and} \quad v_c = \omega \cdot OC,$$

$$v_K = \frac{DK}{DC} v_c, \qquad v_c = \frac{DL}{DC} v_c,$$

also,

$$v_K \cos \alpha = \frac{DK}{DC} v_c \cos \alpha = \frac{DK}{DC} p\omega'.$$

The equation of virtual work is:

$$P \cdot v_K \cos \alpha - F_c v_L + C_B \omega' - F_B b \omega' - F_A v_E - C_A \omega - Q v_A = 0.$$

Substituting the kinematic relations, the value of P can be expressed:

$$P = \frac{DC}{p \cdot DK}\left[F_c \cdot \frac{DL}{DC} \cdot DC - C_B + F_B \cdot b + \frac{OB}{IB}(Q \cdot AI + F_a \cdot IE + C_a) \right],$$

where $\overline{F}_c$, $\overline{F}_B$, $\overline{F}_A$ and C_B, C_A are the kinetic reactions and couples previously listed in the discussion of the same problem by the method of moments.

As an example of the method of work and energy applied to a simple machine and also applied for the calculation of the internal reaction, consider the worm wheel drive, Fig. 66.

The equation of work is

$$P \cdot 2\pi p = T \frac{h}{2\pi R} \cdot 2\pi + \frac{\mu N}{\sin \theta} h,$$

where h = worm pitch, N = normal reaction on worm thread

$$\therefore \quad Pp = \frac{T}{R} r \tan \theta + \frac{\mu N}{\cos \theta} r,$$

where r is the radius of the worm, R = radius of gear.

To compute N, we note:

$$T = ZR = N(\cos \theta - \mu \sin \theta)R,$$

$$\therefore \quad N = \frac{T}{R(\cos \theta - \mu \sin \theta)}.$$

Substituting:

$$Pp = \frac{T}{R} \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} r.$$

PRINCIPLE OF SUPERPOSITION OF REACTIONS.

We have seen so far as an individual member is concerned, any applied or inertia loading may be balanced at the joints by components which must be partly determined by the external conditions. We may superimpose any additional reactions at the joints which form a static balance. The actual reaction at the joint is the resultant of the static and dynamic loadings.

The principle may be extended to any combination of members. In general any applied force must be equilibrated throughout the assemblage of linkages. Each applied force sets up a system of internal reactions throughout the mechanism. If we superimpose the system of reactions set up by the applied forces, we have the resultant or the internal reactions in the mechanism. This resolution is based on the kinetic equilibrium of the members or the system as a whole, so that the reactions set up by each applied force must form an equilibriating system for each member as well as for the system as a whole. We are at liberty further in choosing any resolution provided this condition is met. The resultant reactions due to the external conditions will necessarily be an invariant, irrespective of the choice of the combination of the components.

We may proceed in the force analysis of a mechanism:

1. By an analysis of the total forces and reactions.

2. By a separation of the individual forces or certain combination groups as the inertia loads and the static applied forces. Each loading or group of loadings sets up its own system of reactions. The resultant reactions are then the vector sum of the components corresponding to the different loadings.

THE TURNING BLOCK LINKAGE, FIG. 67.

As an example of the analysis of the reactions consider a turning block linkage, as in the Whitworth Quick Return Motion. Let the torque of the driving shaft on the driving crank "r" be P_θ and the resistance at the driven point "d" along the upper slide be P. The driving crank, r, has a moment of inertia I_r about the shaft. It drives the oscillating lever, l, of moment of inertia I_l about its fixed axis "a," and mass m_l with center of gravity at l_g from "a," by means of a moving sliding block "b," of mass m_b. The connecting rod l' connects the sliding block "d" of mass m_d with the oscillating lever at "c." Its moment of inertia is I_l' about c, and its mass is m_l' with its c. of g. l_g' from c. The crank angle is θ, and the oscillating lever makes an angle ϕ with respect to the vertical, while the connecting rod $l' = cd$, makes an angle ψ with the horizontal.

The geometrical relations are:

$$x_b = r \sin \theta, \qquad x_c = l \sin \phi,$$
$$y_b = h + r \cos \theta, \qquad y_c = l \cos \phi,$$

and

$$x_d = l \sin \phi - l' \cos x,$$
$$y_d = l \cos \phi + l' \sin x = \text{constant}.$$

The distance from the axis "a" along the oscillating lever "l" to the moving sliding block "b" is

$$l_{ab} = \sqrt{h^2 + r^2 + 2hr \cos \theta},$$

so that l_{ab} is a function of θ.

The kinematic condition is

$$l_{ab} \dot{\phi} = r\dot{\theta} \cos (\theta - \psi).$$

The velocity ratios and their derivatives are

$$K_{x\theta\theta} = r\cos\theta, \quad K_{\theta\phi} = \frac{r}{l_{ab}}\cos(\theta-\phi), \; K_{xc\theta} = l\cos\phi K_{\phi\theta},$$

$$K_{y\theta\theta} = -r\sin\theta, \; K_{\psi\theta} = \frac{l\sin\phi}{l'\cos\psi}K_{\phi\theta}, \quad K_{yc\theta} = -l\sin\phi K_{\phi\theta}$$

$$\therefore \quad K_{\psi\theta} = \frac{rl}{l'l_{ab}}\left(\frac{\cos(\theta-\phi)\sin\phi}{\cos\psi}\right),$$

$$K_{xc\theta} = \frac{lr}{l_{ab}}\cos\phi\cos(\theta-\phi), \quad K_{yc\theta} = \frac{-lr}{l_{ab}}\sin\phi\cos(\theta-\phi),$$

$$K_{xd\theta} = \frac{l\cos(\phi-\psi)}{\cos\psi}\, K_{\phi\theta} = \frac{lr}{l_{ab}}\,\frac{\cos(\theta+\psi)\cos(\theta-\phi)}{\cos\psi},$$

$$\frac{dK_{x\theta\theta}}{d\theta} = -r\sin\theta, \qquad \frac{dK_{y\theta\theta}}{d\theta} = -r\cos\theta,$$

$$\frac{dK_{\phi\theta}}{d\theta} = -\frac{r}{l_{ab}}\sin(\theta-\phi)(1-K_{\phi\theta}) - \frac{r}{{l_{ab}}^2}\cos(\theta-\phi)\frac{dl_{ab}}{d\theta}$$

$$= -\frac{r}{l_{ab}}\sin(\theta-\phi)\left(1-\frac{r}{l_{ab}}\cos(\theta-\phi)\right) + \frac{hr^2}{{l_{ab}}^3}\sin\theta\cos(\theta-\phi),$$

$$\frac{dK_{xc\theta}}{d\theta} = -l\sin\phi\cdot{K_{\phi\theta}}^2 + l\cos\phi\frac{dK_{\phi\theta}}{d\theta},$$

$$\frac{dK_{yc\theta}}{d\theta} = -l\cos\phi\cdot{K_{\phi\theta}}^2 - l\sin\phi\frac{dK_{\phi\theta}}{d\theta},$$

$$\frac{dK_{\psi\theta}}{d\theta} = -\frac{l\sin\phi}{l'\cos\psi} = \frac{dK_{\phi\theta}}{d\theta} - \frac{l\cos\phi}{l'\cos\psi}\cdot{K_{\phi\theta}}^2 - \frac{l\sin\phi}{l'\cos^2\psi}\cdot\sin\psi\cdot K_{\psi\theta}\cdot K_{\phi\theta},$$

$$\frac{dK_{xd\theta}}{d\theta} = \frac{l\cos(\phi+\psi)}{\cos\psi}\cdot\frac{dK_{\phi\theta}}{d\theta} + \frac{l\cos(\phi+\psi)}{\cos^2\psi}\cdot\sin\psi\cdot K_{\psi\theta}\cdot K_{\phi\theta} - \frac{l\sin(\phi+\psi)}{\cos\psi}(K_{\phi\theta}+K_{\psi\theta})K_{\phi\theta},$$

where we note,

$$l_{ab} = \sqrt{h^2 + r^2 + 2hr\cos\phi}.$$

For the relative velocity ratio of the moving block along the slide of the oscillating lever, we have

$$r\dot{\theta}\sin(\theta-\phi)=v_{\text{rel}},\qquad\therefore\quad K_{\text{rel}}=r\sin(\theta-\phi).$$

The kinetic energy of the system is,

$$T=\tfrac{1}{2}[I_r\dot{\theta}^2+I_l\dot{\phi}^2+I_l'\dot{\psi}^2+m_l'(\dot{x}_c^2+\dot{y}_c^2+2l_g'(\ddot{y}_c\cos\psi+\dot{x}_c\sin\psi)\psi)+m_d\dot{x}_d^2+m_br^2\dot{\theta}^2]$$

or in terms of velocity ratios, $T=\frac{1}{2}A\dot{\theta}^2$, where the coefficient of inertia A, is,

$$A=[I_r+I_lK_{\phi\theta}^2+I_l'K_{\psi\theta}^2+m_l'(K_{xc\theta}^2+K_{yc\theta}^2+2l_g'(K_{yc\theta}\cos\psi+K_{xc\theta}\sin\psi)K_{\psi\theta})+m_dK_{xd\theta}^2+m_br^2].$$

In the generalized equation of motion we will consider the internal friction at the moving sliding block "b." The reduced work forces are, therefore, neglecting gravity,

$$-P_\theta+PK_{xd\theta}-\mu NK_{\text{rel}\,\theta},\qquad(\mu=\text{coeff. of friction}),$$

where N is the normal reaction between the block and oscillating lever. To determine N, consider the reactions on the block and crank, Fig. 67.

If X_b and Y_b are the component reactions between the crank pin and trunnion of the block, then for the angular motion of the crank,

$$-P_\theta-X_br\cos\theta+Y_br\sin\theta=I_r\ddot{\theta}$$

and for the motion of the block,

$$X_b+\mu N\sin\phi-N\cos\phi=m_b\ddot{x}_b,$$
$$Y_b+\mu N\cos\phi+N\sin\phi=m_b\ddot{y}_b.$$

Combining these equations with that for the crank and noting,

$$\ddot{x}_b=K_{xb\theta}\ddot{\theta}+\frac{dK_{xb\theta}}{d\theta}\dot{\theta}^2=r\cos\theta\cdot\ddot{\theta}-r\sin\theta\cdot\dot{\theta}^2,$$

$$\ddot{y}_b=K_{yb\theta}\ddot{\theta}+\frac{dK_{yb\theta}}{d\theta}\dot{\theta}^2=-r\sin\theta\cdot\ddot{\theta}-r\cos\theta\cdot\dot{\theta}^2,$$

we have,

$$-P_\theta-Nr\cos(\theta-\phi)-\mu Nr\sin(\theta-\phi)=(m_br^2+I_r)\ddot{\theta},$$

which obviously could have been obtained directly by con-

sidering the crank and block together. Therefore,

$$N = \frac{-P_\theta - (m_b r^2 + I_r)\ddot{\theta}}{r(\cos(\theta - \phi) + \mu \sin(\theta - \phi))}.$$

The generalized equation of motion is

$$\left[A - \frac{(m_b r^2 + I_r)K_{\text{rel }\theta}}{r(\cos(\theta - \phi) + \mu \sin(\theta - \phi))}\right]\ddot{\theta}$$
$$+ \left[I_l K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} + I_l' K_{\psi\theta}\frac{dK_{\psi\theta}}{d\theta} + m_l'\left\{ K_{xc\theta}\frac{dK_{xc\theta}}{d\theta}\right.\right.$$
$$+ K_{yc\theta}\frac{dK_{yc\theta}}{d\theta} + l_g'(K_{yc\theta}\cos\psi + K_{xc\theta}\sin\psi)\frac{dK_{\psi\theta}}{d\theta}$$
$$+ l_g'\left(\frac{dK_{yc\theta}}{d\theta}\cos\psi + \frac{dK_{xc\theta}}{d\theta}\sin\psi\right)K_{\psi\theta}$$
$$\left.\left. - l_g'(K_{yc\theta}\sin\psi - K_{xc\theta}\cos\psi)K_{\psi\theta}^2 + m_d K_{xd\theta}\frac{dK_{xd\theta}}{d\theta}\right]\dot{\theta}^2\right.$$
$$= -P_\theta\left[1 - \frac{\mu K_{\text{rel }\theta}}{r(\cos(\theta - \phi) + \mu \sin(\theta - \phi))}\right] + PK_{xd\theta}.$$

Substituting the value $K_{\text{rel }\theta} = r \sin(\theta - \phi)$, we have finally,

$$\left[\frac{(I_r + m_b r^2)\cos(\theta - \phi)}{\cos(\theta - \phi) + \mu \sin(\theta - \phi)} + I_l K_{\phi\theta}^2 + I_l' K_{\psi\theta}^2\right.$$
$$+ m_l'(K_{xc\theta}^2 + K_{yc\theta}^2 + 2l_g'(K_{yc\theta}\cos\psi + K_{xc\theta}\sin\psi)K_{\psi\theta}$$
$$\left. + m_d K_{xd\theta}^2\right]\ddot{\theta} + \left[I_l K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} + I_l' K_{\psi\theta}\frac{dK_{\psi\theta}}{d\theta}\right.$$
$$+ m_l'\left\{ K_{xc\theta}\frac{dK_{xc\theta}}{d\theta} + K_{yc\theta}\frac{dK_{yc\theta}}{d\theta} + l_g'(K_{yc\theta}\cos\psi\right.$$
$$+ K_{xc\theta}\sin\psi)\frac{dK_{\psi\theta}}{d\theta} + l_g'\left(\frac{dK_{yc\theta}}{d\theta}\cdot\cos\psi + \frac{dK_{xc\theta}}{d\theta}\sin\psi\right)K_{\psi\theta}$$
$$\left.\left. - l_g'(K_{yc\theta}\sin\psi - K_{xc\theta}\cos\psi)K_{\psi\theta}^2\right\} + m_d K_{xd\theta}\frac{dK_{xd\theta}}{d\theta}\right]\dot{\theta}^2$$
$$= -\frac{P_\theta\cos(\theta - \phi)}{\cos(\theta - \phi) + \mu \sin(\theta - \phi)} + PK_{xd\theta}.$$

Substituting the velocity ratios previously derived, we obtain the equation of motion in terms of the configuration of the system and the principle coördinate.

(To be continued.)

Experiments on the Apparent Deviation from Ohm's Law for Metals at High Current Densities. H. M. BARLOW. (*Phil. Mag.*, May, 1930.) Several investigations of the validity of Ohm's Law for high current densities in metals have been made but all with one exception indicate that any departure that may possibly exist is exceedingly small even for 10,000,000 amperes per sq. cm. "Professor Bridgman, however, claims to have observed and measured a definite increase in the effective resistance of gold and silver films to currents of the order of 10^6 amp. per sq. cm., and this after eliminating errors due to rise of temperature." He published his results in 1921 and 1922. In his investigation he passed a strong direct current and a weak alternating current of audible frequency at the same time through a metallic film. "The P.D. between the ends of the film was balanced in a simple bridge circuit first with a moving coil galvanometer and then with a pair of telephones. The balancing resistances were exactly the same for the alternating current and the direct current until the density reached about 10^5 or 10^6 amp. per sq. cm., but after this small differences were observed." As the resistance in ohms is for direct currents equal to the volts divided by the amperes this points to a change in resistance, which persisted even after careful elimination of disturbing factors. Barlow pursued the same experimental method, introducing certain refinements to get greater accuracy. He does not confirm Bridgman's findings but concludes "The foregoing observations prove that Ohm's law is rigidly true for a gold conductor carrying a current of density 2×10^6 amp. per sq. cm., and leave little room for doubt that the law is equally true at 3×10^6 amp. per sq. cm. The same conclusion is indicated for platinum at a current density of 1.3×10^5 amp. per sq. cm."

G. F. S.

COMMITTEE ON SCIENCE AND THE ARTS.

ABSTRACTS OF REPORTS.

WATERLESS GAS HOLDER.

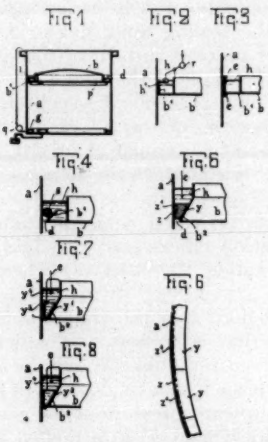
HALL OF THE INSTITUTE,
PHILADELPHIA, May 2, 1928.

No. 2859.

The Franklin Institute of the State of Pennsylvania acting through its Committee on Science and the Arts, investigating the Waterless Gas Holder, invented by Mr. Konrad Jagschitz, of Mainz, Germany, reports as follows:

As hardly need be stated, the gas holder universally employed prior to this invention is the water sealed type.

PLATE I



The purpose of this invention is to provide a gas reservoir in which the gas is contained underneath a disk cover which is capable of vertical movement, its position being determined by the amount of gas in the reservoir. The disk carries at

its outer edge a shallow trough containing a thick oil or tar which on trickling down the side of the reservoir forms an effective seal between the disk and the side.

A number of United States Patents have been granted on this device to Konrad Jagschitz, of Mainz, Germany, the fundamental patent being No. 1,275,696 granted on August 13, 1918 on an application filed on July 31, 1914. In addition to the basic features of design this patent covers numerous details of construction, methods of erecting the structure, etc., and although naturally subject to modification with experience, these details are largely employed in the latest construction. It is the basic features of design, however, which are of chief interest.

Plate I is made from drawings in this patent. Fig. *1* is a vertical section of the reservoir in its simplest form. In this figure, *a* is the reservoir in which the disk shaped cover *b* is movable vertically as the volume of the gas changes.

Around the outside of *b* it is formed into a trough *b'* which contains the sealing fluid *h*, Fig. *2*, enough of which engages the inner vertical surface of the reservoir to form a seal, preventing leakage of the gas. The fluid that passes through *d* and down the wall *a* is collected in the annular trough *g* at the bottom of the reservoir. A pump *q* is used to raise this fluid to the annular channel *i*, at the upper edge of the reservoir, from which it is again passed into the trough *b'*.

The cover *b* is provided with an annular flange *p* that in its lowest position enters into the fluid contained in the trough *g*, thus sealing the reservoir when it is practically empty.

The height of fluid *h* in the trough *b'* is dependent upon the gas pressure that is present. In order to maintain the flow of a small amount of fluid through the space *d*, the height of the fluid is made such that its downward pressure is slightly greater than the upward pressure of the gas.

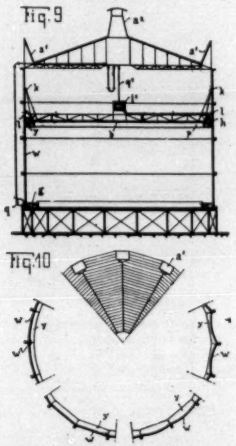
Figure *2* shows a float *h'* so arranged as to regulate the rate of flow of the sealing fluid through valve *r* that controls the flow back from the trough at the bottom of the reservoir. Various devices are used to reduce the flow of the sealing fluid to a minimum. Fig. *3* illustrates a form in which sealing strips *E* of jute or any suitable packing material are carried

by the trough b' and slide along the inner surface of the reservoir.

In Fig. *4* the wall b' of the trough is inclined towards the wall of the reservoir and an annular sealing roll s is placed in the space between the walls. In Figs. *5* and *6* the trough is omitted and the outer wall of the cover disk is inclined towards the wall of the reservoir as shown. Wedged shaped pieces of sealing material are placed end to end at y and are kept in place by the pressure of the fluid C and form a sealing ring. Channels z are formed in the outer side to contain a portion of the sealing fluid.

Figure *7* shows a form in which the fluid is contained in a trough of wedged shaped cross section, the seal having an extension along its upper edge as at y^3, that slides along the wall of the reservoir. Figure *8* is a modification in which the outer edge is smooth and its contact with the reservoir wall is regulated by the pressure of a spring support y^4.

PLATE II



Figures *9* and *10*, Plate II, illustrate a reservoir constructed of flanged plates and covered by a roof. In this form the

receiving channel i is replaced by a tank i' carried by the cover and from which the fluid is delivered to the sealing troughs. Rollers k and i guide the vertical movements of the cover, travelling along either the inner surface of the plates W or along strengthening rails W'. Extensions a' are provided to receive the upper rollers k when the reservoir is filled with gas.

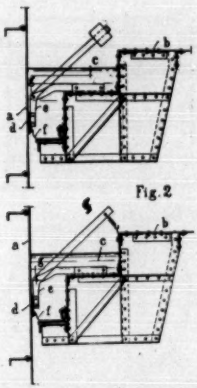
Figure 10 illustrates the employment of differently shaped plates for the reservoir wall, either circular or arched and the method of joining them to form an effective seal.

Plate II shows a waterless gas holder in both vertical and horizontal section.

Since the value of this device depends upon the degree to which the leakage of gas from the holder is controlled, the seal between the cover and the inner wall is an important feature. An improved form of seal is described in United States Patent No. 1,481,099 from drawings in which Plate III is made.

PLATE III

Fig 1

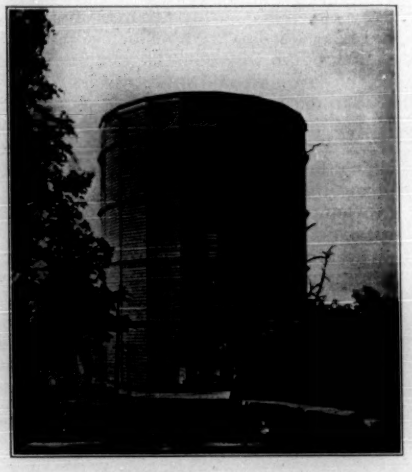


In Fig. 1, which is in vertical section, a represents the wall of the gas holder along the inside of which the cover b with its seal d is vertically movable. The trough c contains the

sealing liquid while a packing *f* helps to control the flow of the sealing liquid down the wall of the holder. The seal *d* is pressed against the wall of the holder by means of a weighted lever pivoted at *g*, the position of the weight on the lever determining the pressure applied. A second form in which a spring replaces the weight is shown in Fig. *2*.

Plate IV shows an external view of this holder.

PLATE IV



Despite the somewhat apparent advantages of this method of construction, its proposal by the inventor seems to have been essentially new. During the entire existence of the gas business, up to the date of this invention, the water sealed type of holder had been practically the only one known or used; its use had become so well seated and had been on the whole so very successful, that it was difficult for any competitive construction to be conceived and adopted. While the advantages of the waterless type may have possibly crossed the minds of some engineers, apparently the uncertainties of this practical construction and the established

condition of the water sealed type of holder, prevented any definite proposal of the use of the waterless type.

The original patent No. 1,275,696—August 13, 1918—Konrad Jagschitz, in its specification and claims (23) goes very largely, as indicated by the foregoing abstract of the specification, into details as to the piston seal, methods of construction, etc. Its feature of especial interest however consists in the first three claims which are quoted as follows:

1. "The combination of a reservoir, a closure slidable along the inner surface thereof and a fluid seal in said reservoir having no effective portion below the sealing point whereby the joint is sealed between said closure and said inner surface.
2. "The combination of a reservoir, a closure movable therein and a fluid sealing medium carried entirely by said closure and having no effective portion below the sealing point, said sealing medium engaging the inner surface of said reservoir whereby the joint between said closure and said inner surface is sealed.
3. "The combination of a reservoir, a closure movable therein, an annular peripheral trough carried by said closure and slidable along the inner surface of said reservoir, and a fluid sealing medium carried entirely by said trough and engaging the inner surface of said reservoir whereby the joint between said closure and said inner surface is sealed."

These claims are very broad. They are limited chiefly by the specification of a fluid seal, or a fluid sealing medium, but as far as can be now foreseen, this is the only available method of rendering this construction safe and practical. (The clause, in claim 1, "a fluid seal in said reservoir having no effective portion below the sealing point" differentiates this construction from the water tank seal, and the telescopic seal of the ordinary water sealed holder.)

In spite of its comparatively wide scope, this patent encountered very little opposition in the United States Patent Office. We have searched the file wrapper and it disclosed no citation which anticipates the above claims.

While the precise breadth of the terms "fluid seal" and "fluid sealing medium" might possibly be somewhat un-

certain in interpretation by a court, it is the opinion of a very excellent patent attorney that, in view of the general novelty of this invention and its practical development, these terms would be interpreted by a court broadly in favor of the patentee.

The chief advantages of the waterless construction, as compared with the ordinary water sealed holder, may be briefly outlined as follows:

The water sealed holder demands a tank filled with water, and in the larger sizes of holders this tank may rise to 40 feet depth of water; this evidently demands very heavy foundation—even when built on excellent ground, the entire tank must be underlaid with a comparatively heavy concrete foundation, and if the ground is bad, expensive piling is required beneath the entire tank bottom. This great weight of water not being required with the waterless holder, the entire foundation expense is very greatly lessened; strong foundation or piling, is really only required beneath the circumference of the holder. And although the tanks of the water sealed holder (steel overground) are in the vast majority of cases entirely satisfactory as regards strength and tightness, serious accident has been known in the past due to the rupturing of a tank.

The telescopic water seal of the water sealed holder requires heating in cold weather, and will be, of course, subject to considerable care with respect to snow, etc., stoppage. Also in the water sealed holder the tank itself requires heating. These requirements which cause expense and necessitate more or less risk, are avoided or lessened in the waterless holder.

In the water sealed type of holder there is continual rising and falling of the holder bell in the water tank. This seriously effects the external surface of the holder, tending toward oxidation and necessitating frequent painting. This is entirely avoided in the waterless type.

As generally constructed, the gas pressure of the water sealed type of holder will necessarily vary as the successive telescopic sections of the holder come into use in the rise of the holder or go out of use as it descends. The pressure given by the waterless type is on the contrary, practically uniform throughout its entire range of capacity.

The waterless type of holder is capable of greater capacity per unit of ground area than the water sealed type.

Inspection of practically the entire waterless holder (internal and external) is to a large degree possible during its working, and any special inspection is very much easier than with the water sealed type.

The total weight of material (steel) required for a waterless holder of given capacity is very markedly lower than for the water sealed type.

It is appearing increasingly desirable to gas engineers to distribute a gas of low water vapor content; the deposition or absorption of this vapor in mains, and more especially meters, tends to deterioration of this equipment. Dehydration of the gas is desirable. Such dehydration will generally be effected before storage of gas in the holder and in the case of a water tank holder it will be difficult to prevent the absorption of water by the gas; with the waterless type holder, this danger does not exist.

In consideration of the technical merits of his inventions which have been proved by practical use, THE FRANKLIN INSTITUTE awards its EDWARD LONGSTRETH MEDAL to KONRAD JAGSCHITZ, of Mainz, Germany.

NOTES FROM U. S. BUREAU OF STANDARDS.*

FIRE PREVENTION AND PROTECTION IN GOVERNMENT BUILDINGS.

Recent fires in Government buildings caused considerable newspaper comment relative to the degree to which precautions are being taken to provide safety for the buildings, their occupants, record storage, and other valuable contents. While the fire record for Government buildings has been in general comparatively favorable, it is known that hazards exist in them that may cause loss of life, valuable records, and damage to structures that often are of monumental type. Realizing this situation representatives of the different departments and establishments at a meeting called by the Secretary of Commerce formed an organization to function in an informative and advisory capacity on matters of fire prevention and protection arising in connection with Government activities. This includes such subjects as exit requirements in buildings, protection of records, elimination of fire hazards, fire detecting and extinguishing equipment, and fire alarm and patrol service, in so far as not covered by other interdepartmental agencies. It was further decided that the given organization should serve as a means of contact between the agencies within the different Government units that are charged with similar activities.

The group, later organized as the Federal Fire Council with the Director of the Bureau of Standards as chairman, functions through committees of which one on fire hazards and another on apparatus and devices have been organized. Committees on inspection and interdepartmental coöperation, and on fire record have been authorized but not as yet fully organized. While the period since the first meeting last April has been mainly given over to organization and other preparatory details several requests for assistance have received attention. These include a preliminary survey of twelve institutions under the control of the Board of Public Welfare of the Dis-

* Communicated by the Director.

trict of Columbia, with report giving recommendations for structural betterment and installation of fire protective equipment. Consideration was also given to the protection required for the Patent Office records in the new Commerce Building.

FURNACE FOR FIRE TESTS OF WALLS AND PARTITIONS.

A new furnace for conducting fire tests of wall and partition constructions has been completed to replace one that had been in use for a number of years. An appropriation of $22,000 was granted for moving the structural framing of the old shelter and furnace to the new location and for their reerection with suitable enclosure. The walls, roof and floor of the new shelter are of reinforced concrete designed to withstand accidental explosive pressure from within of 600 lbs./ft.^{2} before failure occurs. Window openings on three sides, filled with steel sash and plain glass, cover about 30 per cent. of the wall area and it is expected that the window construction will be forced out before pressures of 60 lbs./ft.^{2} are developed, and thus relieve the pressure within the building before attaining intensities that would cause collapse of main structural members. The fuel used in the furnace is gas and while safety measures have been incorporated in the installation to make remote occurrence of explosive mixtures or explosions within the enclosure and the furnace, it was nevertheless deemed advisable to provide the given structural features. The added cost over ordinary construction was quite small and was incurred mainly in properly disposing the reinforcing steel, providing brackets on beams and wide fillets or splays on the interior sections of columns to give the needed tensional resistance. Tensional resistance in the opposite direction must also be provided to resist stresses from rebound after high pressure from the inside. Provision must also be made to resist the uplift by means of floor reinforcement and proper design of foundation.

That extreme proportions of members are not required to attain the higher resistance outlined above is shown by the fact that walls are only 8 inches thick for center to center distance between columns of 26 feet, and the roof slab is 4 inches thick supported on 12 by 14 inch beams, 5 feet apart, and

spanning a maximum distance of 29 feet 8 inches, center to center of trusses. It is believed that similar principles of design can be applied to advantage in designing commercial and industrial buildings in which explosive pressures may occur. The life loss from such explosions is caused more generally by the accompanying building collapse than by direct temperature and force effects. Wood, masonry, and concrete buildings as ordinarily constructed can generally not withstand pressures over 100 lbs./ft.^{2} before collapse occurs.

The furnace is designed to accomodate specimens 10 to 11 feet high, and 16 feet wide. The walls or partitions to be tested are built within heavy moveable frames and are placed for the fire test so as to form one side of the furnace chamber. Partitions and walls are tested either restrained and non-bearing to simulate conditions where they are built between heavy columns and floor members in fire resistive buildings, or they are tested under nominal working loads. In the present equipment, the load is applied by means of four hydraulic jacks resting on the bottom member of the restraining frame and supporting a steel member on which the construction to be tested is built. Loads up to 400,000 lbs. can be applied before, during, or after the fire, or fire and water test. In addition, heavy walls are tested unrestrained, a condition obtained by leaving a space at the sides and top between the containing frame and the wall to be tested. This allows the wall full freedom to expand and deflect and simulates conditions for walls in the top story, and also in lower stories where the interior framing is light and not anchored to the wall.

The fire exposure is obtained by means of 92 venturi tube induction type burners distributed over the back of the chamber opposite the wall to be tested. The design of these burners was developed for the requirements presented by this furnace, which include uniformity of temperature over the test wall with absence of directly impinging flames. In these burners the air required for combustion is drawn in at the cold end of the burner by the gas jet without aid of air compresser or fan. The large gas consumption, 16,000 to 20,000 cubic feet per hour, necessitated a new gas connection to the street main and a gas booster in order that this amount might be delivered to the burners at the required pressure.

SEAMS FOR COPPER ROOFING.

Sheet copper has been used as a roofing material for hundreds of years. In the course of this time, some five or six types of seams for joining the sheets have been proved by experience to be suitable for copper roofing. Two of these types formed the main subject of an investigation recently completed by the bureau.

On many old buildings with copper roofs the seams are calked with a white lead and linseed oil mixture. A notable example is the State House in Boston. Although these calked seams have given very satisfactory service, an exact measure of their water-proof qualities has not previously been made. Leakage tests on both new and old seams showed that this type of seam can be used wherever the depth of water which may remain standing for several days on the roof will not exceed four inches. This depth rarely occurs except in gutters. A small section of seam removed from a roof after many years of service stood continuously for 45 days under 12 inches of water without any signs of leaking.

Many divergent opinions concerning soldered seams are held by experienced roofers. An elaborate series of tensile tests on 800 specimens was made in order to settle the question in issue and show how to obtain the strongest seams. Most roofers will use only killed acid flux. While this may possibly be justified from the standpoint of convenience in soldering, the tests showed that resin flux and a prepared commercial flux gave equally strong seams. Perhaps the most disputed point is the question whether flat lock seams require large amounts of solder on the upper surface in order to give the maximum strength. It was found that the strength was increased by using larger quantities of solder, but the increase was slight. However, by pretinning, that is, dipping the edges of the sheets to be joined in flux and then in molten solder or tin, the maximum strength was obtained. Pretinned seams were very much stronger than those not pretinned even though made with large amounts of solder.

Another question is the effect of continuous load on a soldered seam. This was studied by suspending heavy loads from small sections of seams for periods of less than one to nearly 300 days. The results indicate that a soldered lap

seam can sustain a load of about 350 lbs./in.^{2} of seam indefinitely without failure. The limit for pretinned 1/2-inch flat lock seams was found to be about 375 lbs./in.^{2} These values can be employed to design seams for the conditions under which they are to be used. The bureau recommends that a factor of safety be adopted since it is not possible properly to inspect a finished seam.

This information will be more fully described in the September number of the Bureau of Standards Journal of Research.

WIND PRESSURE ON CHIMNEYS.

One of the major elements determining the cost of a large chimney or stack is the amount of wind pressure which it is to withstand. Many specifications are drawn up in which the chimney is required to withstand a wind of 100 miles per hour, but since there are all sorts of opinions and little knowledge as to the pressures imposed by a wind of this amount, different bidders often design for different loads and therefore do not bid on structures of the same strength. In many instances it is felt that the values of wind pressure used are too high, thus increasing the cost without corresponding benefit.

For several years, an investigation of the wind pressure on chimneys and other cylindrical structures has been in progress at the bureau. Model testing in wind tunnels does not give a satisfactory answer in this instance, and it has been necessary to supplement these tests by measurements on large structures in natural winds. The investigation showed that the wind pressure is a function of the ratio of the height of the chimney to its diameter and that a wind pressure corresponding to 20 lbs./ft.^{2} of projected area at a wind speed of 100 miles per hour is a safe value to use in designing chimneys of which the exposed height does not exceed ten times the diameter.

This investigation will be discussed at greater length in the September number of the Bureau of Standards Journal of Research.

CHANGES IN VOLUME OF BURNED CLAY BODIES.

The Columbus Branch of the bureau is conducting an investigation of the change in volume of burned clay bodies resulting from the autoclave treatment, and is comparing this change with that produced by actual weathering outdoors and storing in a damp jar over water. Five bars of each of several commercially fired bodies were subjected to a three-hour autoclave treatment under a steam pressure of 150 lbs./in.^{2} and their changes in length noted. The absorptions of these bars were determined after the autoclave treatment. A corresponding set of five bars of each body was placed on a single story roof outdoors and another set was placed in a closed container over water. The changes in length of these bars as recorded after the first six months (from June to December) and after the second six months (from December to June) are given below with each value representing the averages of the five bars.

A buff wall tile body expanded 0.057 per cent. in the autoclave; 0.016 per cent. in six months outdoors; 0.009 per cent. in six months in a damp jar; 0.032 per cent. in twelve months outdoors; and 0.025 per cent. in twelve months in a damp jar; and had an absorption of 6.6 per cent.

A white wall tile body expanded 0.069 per cent. in the autoclave; 0.053 per cent. in six months outdoors; 0.053 per cent. in six months in a damp jar; 0.053 per cent. in twelve months outdoors; and 0.070 per cent. in twelve months in a damp jar; and had an absorption of 15.0 per cent.

A soft fired terra cotta body expanded 0.056 per cent. in the autoclave; 0.030 per cent. in six months outdoors; 0.025 per cent. in six months in a damp jar; 0.034 per cent. in twelve months outdoors; and 0.043 per cent. in twelve months in a damp jar; and had an absorption of 12.5 per cent.

A white semi-vitreous dinnerware body expanded 0.071 per cent. in the autoclave; 0.063 per cent. in six months outdoors; 0.053 per cent. in six months in a damp jar; 0.072 per cent. in twelve months outdoors; and 0.076 per cent. in twelve months in a damp jar; and had an absorption of 9.6 per cent.

An ivory semi-vitreous dinnerware body expanded 0.065 per cent. in the autoclave; 0.063 per cent. in six months outdoors; 0.048 per cent. in six months in a damp jar; 0.086 per

cent. in twelve months outdoors; and 0.078 per cent. in twelve months in a damp jar; and had an absorption of 7.8 per cent.

EFFECT OF VARIATIONS IN COMPOSITION ON PROPERTIES OF VITREOUS ENAMELS.

It is customary to express the compositions of enamels in terms of the percentages of the various constituents (mostly oxides), no attempt being made to indicate the actual chemical constitution, for lack of knowledge on that point. There is considerable evidence, however, that enamels which have virtually identical ultimate chemical compositions may sometimes differ markedly in properties, provided different combinations of raw materials are used. Thus the sodium content of an enamel, customarily expressed as Na_2O, may be derived from the carbonate and the nitrate of soda, borax, feldspar, and other sodium-bearing raw materials. Obviously, the proportions in which these materials are used may be varied without disturbing the resultant sodium content of the enamel.

It was considered important to learn something more definite about the effects of variations in raw materials on the properties of enamels of given composition before attempting to correlate the compositions with quantatively determined properties. Using analyzed raw materials, two enamels were prepared having the same calculated ultimate composition but different batch compositions. Cone deformation tests showed a distinct difference in fusibility between the two enamels. There was also a detectable difference in the indices of refraction. To indicate whether these differences might be due to a state of arrested reaction in one or both of the enamels, additional heat treatment was given in platinum crucibles at the temperature of preparation (1250° C.). The initial melting occupied about one hour, and the additional treatment was carried out over periods of 4, 6, and 12 hours, respectively. The cone deformation temperature of each enamel remained virtually unchanged throughout these tests, and the indices of refraction were still different at the end of the 12-hour treatment. Hence, the difference in properties between the two enamels appears to be due to some more stable condition than mere arrested reaction. The compositions of the differently

treated frits as determined by chemical analysis, will have to be considered before drawing final conclusions.

THERMAL EXPANSION OF CHROMITE.

Several samples of chrome ore are included in the bureau's investigation of the linear thermal expansion of special refractories to 1800° C. Some data on expansion of Imperial (Rhodesian), African (friable), Grecian, and Indian chrome ores have been obtained. These ores all have the same characteristic expansion, namely, when tested from 20° to 1000° C. in a neutral to oxidizing atmosphere, the rate of expansion of specimens fired at cone 23 (approximately 1580° C.) is uniform throughout the entire temperature range, and the total expansion at 1000° C. is between 0.7 and 0.85 per cent. However, under reducing conditions the rate of expansion increases greatly, and in the case of the African ore the increase is enormous. For comparative purposes the following table gives the average coefficient of linear thermal expansion from 20° to 1000° C. of the materials in a (1) neutral to oxidizing atmosphere and in a (2) reducing atmosphere.

Kind of ore.	Kind of atmosphere.	
	Oxidizing.	Reducing.
Imperial	7.29×10^{-6}	17.45×10^{-6}
African	8.51×10^{-6}	39.01×10^{-6}
Grecian	8.32×10^{-6}	10.06×10^{-6}
Indian	7.29×10^{-6}	15.91×10^{-6}

The total expansion of the African ore was 4.8 per cent at 1030° C. (reducing atmosphere). After cooling it was found that the specimen had increased approximately 4.3 per cent. in length.

The specimens of Imperial, Grecian, and Indian ores, which were heated in the testing apparatus to 1800° C. (reducing atmosphere only), showed initial contraction at approximately 1300° C. and contracted rapidly between 1400° and 1800° C. The maximum expansion of the Imperial ore was 1.93 per cent. obtained at 1200° C.; the Grecian ore, 1.16 per cent. obtained also at 1200° C. and the Indian ore 1.83 per cent. obtained at 1300° C.

The specimen of Imperial ore fired at cone 23 and heated to 1800° C. was tested a second time. The maximum expansion, 1.20 per cent. was obtained at 1430° C. Although the specimen began to contract after reaching this temperature and continued to do so to 1800° C., the rate of contraction was decidedly lower than in the initial test. At 1800° C. the total expansion was 0.82 per cent. The average coefficient of expansion from 20° to 1430° C. is 8.5×10^{-6}.

The petrographic examination of these ores after firing at cone 23 (1580° C.) and after heating in a reducing atmosphere either to 1000° C. or to 1800° C. showed no change in composition.

ACCELERATED WEATHERING OF LETTER BOX ENAMELS.

Two samples of olive green letter box enamels were tested for durability for the Post Office Department. One showed bad checking within 14 days, while the other was free from checking or cracking within the same period. In purchasing this enamel, the Post Office Department for the fiscal year beginning July 1, 1930, has adopted and has printed the accelerated method of test as a part of the specifications. This is the first Government department, so far as is known, to use this method of test as a printed requirement of their specifications. They require that a two coat job of enamel, when exposed in the accelerated cycle (as used at the Bureau of Standards) for 14 continuous days, shall show no color fading, checking, cracking, chalking, and not over a slight loss of gloss. The preparation of the panels and the method of test are described in detail. These enamels are pigmented oleoresinous varnishes that are used on letter boxes out of doors. They must be very durable, and a chemical examination of the enamel, especially of the varnish vehicle, is of little use in predicting durability. The 14 day requirement that the coat of enamel shall be free from defects is based on a large number of exposure tests, both outdoors and in the accelerated test. The requirement is reasonably severe, so as to allow only high-grade enamels. In one exposure test, six out of 22 enamels from as many different manufacturers, passed the test.

DETERMINATION OF OSMIUM.

It is believed that the new method for the determination of osmium (mentioned in Technical News Bulletins Nos. 156 and 157), as now completely developed successfully overcomes the difficulties which have heretofore been experienced and offers a simple and accurate means for the determination of this metal.

The method may be briefly described as follows: Osmium tetroxide separated from other metals by distilling with nitric acid is absorbed in 6 *N* hydrochloric acid saturated with sulphur dioxide and contained in two receiving flasks attached to the distilling flask by ground joints. After the distillation is complete the receiving solution is transferred to a beaker and evaporated on the steam bath. The residue, which will not go quite to dryness because of the presence of sulphuric acid formed by oxidation of the sulphur dioxide by the osmium tetroxide and nitric acid which were distilled into the solution, is digested for 15 minutes with a small amount of strong hydrochloric acid. The solution is again evaporated and the digestion with hydrochloric acid repeated. After evaporating once more the residue is diluted with water to about 150 c.c. and the solution heated to boiling. A few drops of brom phenol blue are added and then a dilute solution of sodium bicarbonate until the alkaline color of the indicator appears (about pH 4). At this alkalinity the osmium is completely precipitated as hydrated osmium dioxide, which is well coagulated on boiling for 5 minutes. The precipitate is filtered off on a Gooch or preferably a Munroe crucible and washed with hot 1 per cent. ammonium chloride solution. After the washing is completed the residue is moistened with saturated ammonium chloride solution. This treatment completely prevents the tendency of the dioxide to deflagrate on subsequent ignition. The crucible is covered with a Rose lid and a slow stream of hydrogen introduced. Heating is done cautiously until the ammonium chloride has been volatilized after which the crucible is heated to bright redness. Heating is continued at this temperature for a few minutes after which the crucible is cooled in the current of hydrogen. When the crucible is at room temperature the current of hydrogen is displaced by a current of carbon dioxide, care being taken that no air is admitted. After the

current of carbon dioxide has been passed for a few minutes the crucible is weighed. Osmium is thus weighed as metal and the results obtained with careful work are very precise.

DIMENSIONAL CHANGES IN GLASS RESULTING FROM HEATING CYCLES.

In obtaining expansivity curves for a medium flint glass, samples of the glass were tested which had previously received one or another of several different heat treatments. In these tests heating cycles were employed which made it possible to determine the magnitude of the so-called "permanent" changes in length which are caused either by the annealing or the "disannealing" which may result from the heat treatment imposed on the glass by the heating cycles employed. Annealing decreases the length and specific volume, while disannealing increases them. These effects are related to certain ice-point shifts observed in thermometers, and to other effects sometimes observed in various types of glassware.

This investigation will be described in greater detail in the September number of the Bureau of Standards *Journal of Research*.

WEATHERING OF SPECIAL WINDOW GLASSES.

Over 1½ years ago the bureau called attention to the fact that the special window glasses, which are used to transmit ultra-violet radiation, undergo a greater decrease in transmission when exposed to the mercury arc than when exposed to the sun; and that further exposure of these glasses to the sun, after exposure to the mercury lamp, increases the ultraviolet transmission to a higher value than the minimum attained by prolonged exposure only to the sun. This overshooting in transmission, which ranged from 1 to 6 per cent. above the minimum attained by exposure only to sunlight was found in 17 out of 20 samples of different makes of glass, including the most widely known glasses, such as Vitaglass and Helioglass. It was then indicated that a further investigation would be required to determine whether this condition would remain permanent on further exposure to the hot, summer sun. To this end the exposure of these glasses to the sun was continued

and their transmissions determined in January and March, 1930.

During the past month further measurements were made on these glasses, showing that this overshooting in transmission is not permanent; but increases to a maximum after which it decreases again to a value that is closely the same as, or somewhat lower than that attained by exposure only to the sun.

Experiments show that the decrease in transmission, caused by the action of ultra-violet radiation, can be counteracted by maintaining the glass at suitably high temperature. The rejuvenation tests, with sunlight transmitted through filters, show that wave-lengths in the region of 365 to 400 mμ produce the greatest rejuvenation; but sufficient time has not yet elapsed to determine whether this will remain permanent.

ULTRA-VIOLET RADIATION FROM GAS STOVES.

The ordinary gas stove, in which the clay radiants are heated to incandescence by means of bunsen gas flames, is an excellent source of infra-red radiation. But owing to their low temperature these radiants, even when covered with special materials, claimed to emit ultra-violet, do not emit a sufficient amount of ultra-violet to merit consideration as a source of short wave-length ultra-violet radiation useful for healing purposes.

This was established at the bureau by two independent methods of observation which were amply sensitive to detect ultra-violet radiation, if emitted. By photographing the spectrum, it was established that the intensity of the ultra-violet emitted by a gas stove is less than 1/800,000 that of similar ultra-violet wave-lengths in sky radiation.

By means of a sodium in quartz photoelectric cell, it was found that at a distance of 50 cm. from a radiant heater, the ultra-violet emitted was less than 1/100,000 of the ultra-violet of wave-lengths less than 313 mμ contained in noon hour June sunlight.

VALUE OF ADDITION AGENTS IN COPPER ELECTROTYPING SOLUTIONS.

In the manufacture of copper electrotypes for the printing industry, various addition agents are used in the solutions in order to permit more rapid deposition and to produce harder deposits of copper. A recent investigation at the bureau has shown that of the substances which may be used for this purpose, phenol (carbolic acid) is most effective. When a small amount of this substance (in the form of a compound with sulphuric acid) is added to the solutions, deposits can be produced under favorable conditions in about one-fourth the time commonly used in electrotyping. Such deposits are also harder than those obtained without any addition agents, and therefore will give better service on the printing presses.

This work will be described in detail in the September number of the Bureau of Standards *Journal of Research*.

PAPER-COATING MINERALS AND ADHESIVES.

As a result of the interest manifested among manufacturers in the investigation on "Use of Glue in Coated Paper" (Technologic Paper No. 323) other paper coating materials have recently been studied. The purpose of the investigation was to keep the public informed of new developments in raw materials and promote the use of domestic resources, which would become very important in the event of any disturbance limiting the supplies of foreign origin. In the later study four different coating clays, two foreign and two domestic, and one commercial compound of domestic diatomaceous earth were the minerals employed. Three types of high-grade commercial adhesives—casein, glue, and modified starches—were used. The tests were confined to papers suitable for high-grade printing. The coating procedure was similar to the commercial practice employed in applying a single coating to paper.

The clays worked satisfactorily with the different adhesives, the domestic clays comparing favorably with the foreign, but the coating of diatomaceous earth did not adhere well to the paper. It was believed that the loss of adhesiveness was due to the high alkalinity of the mineral.

Relative to the comparative quality of the adhesives as paper coating materials, the tests indicate that the starches may not have as good adhesive quality as casein or glue, but all coatings containing 18 parts of starch per 100 parts of clay were well bound to the body papers. Graded as to their clay suspending property the adhesives are rated in the following order: starch, casein, and glue.

Printing tests of the papers were made at the Government Printing Office which used their regular half-tone process. Equally good results were obtained with the American and foreign clays, and with the three different kinds of adhesives. Both the starch and glue coatings, however, had less water resistance than the casein coatings, and therefore would possibly not be suitable for lithographic processes requiring a high degree of water resistance.

A complete report of this investigation is being prepared for publication.

THE INFLUENCE OF SPLITTING ON THE STRENGTH OF LEATHER.

In the manufacture and use of leather, splitting the hide in order to secure leather having a uniform and definite thickness is a common practice. Grain upholstery and upper leathers made from steer hides are examples of split leathers, the thickness of which represents from 25 to 35 per cent. of the total thickness of the leather made from the hides.

In many other cases where approximately the full thickness of leather produced is used, it is often necessary to split off certain portions of the flesh fibers in order to obtain an even thickness. The effects of these splitting operations have been studied, and the results appear to explain the failure of some leathers in service.

A typical example is that of a sample of calf upper leather which tore when subjected to the tension of the lasting operations in the shoe factory. Microscopic examination revealed that the leather had been split so as to remove about one-half of the thickness of that originally produced.

Tests made on several samples of calf upper leather showed that grain splits varying in thickness from 20 to 37 per cent. of that of the unsplit portions possessed from 3 1/2 to 12 per cent.

of the breaking strength of the unsplit portions. The breaking strength of the corresponding flesh splits varied from 54 to 70 per cent. of that of the unsplit leathers. The results of the tests demonstrate that grain splits, ranging in thickness up to about 50 per cent. of that of the unsplit leathers, have materially less strength than either the unsplit or flesh split portions of the same leathers. A further serious effect of splitting is that the percentage stretch of grain splits, at a definite stress, is always much greater than for the unsplit leathers.

Although the removal of a certain amount of the strength-giving fibers from the flesh side of leather generally leaves a grain thickness having satisfactory strength properties for the service intended, the manufacturer of leather should maintain a careful check on the operation in order to avoid unnecessary failures of otherwise satisfactory material when used for shoe uppers, straps, bags, upholstering, and transmission belting.

AËRONAUTIC RADIO RESEARCH.

Advantage was taken of the severe atmospherics prevalent at this time to determine the minimum power requirements for the runway localizing beacon transmitter. A distance range of approximately 10 miles is required for this beacon under all conditions of radio reception. A number of tests indicate that this distance range is secured, with some margin to spare, when using only 100 watts in each loop antenna, the corresponding power previously employed being 250 watts. This reduction in power considerably simplifies the transmitting circuit arrangement, and reduced the cost of the localizing beacon installation by fully 25 per cent.

Further improvements were incorporated in the automatic volume control device recently developed by the research division. This device operates to maintain constant receiving set output signal intensity regardless of the magnitude of the input voltage. Its operation depends upon the rectification of the output voltage and the application of this rectified voltage as negative bias on the grids of the radio-frequency amplifying tubes of the receiving set. Any increase in the output voltage due to increasing input voltage is accompanied by an increase in this negative bias. This serves to reduce the sensitivity of

the radio-frequency amplifying tubes, thereby maintaining substantially constant output voltage. Increasing input voltages are thus accompanied by increasing negative biasing voltage on the radio-frequency tubes. A d.c. milliammeter reading plate current supply will therefore show smaller deflections as the receiving set input voltage is increased.

When flying on the visual type radio range beacon, this milliammeter may be used as an approximate distance indicator. As the distance of the airplane from the beacon is reduced the input voltage increases, thereby reducing the plate meter deflection. The reverse is true if the distance of the airplane from the beacon increases. Flight tests made on this device have demonstrated its possibilities, particularly on the runway localizing beacon used in the research division's blind landing system. By virtue of its use as an approximate distance indicator it at once tells the pilot whether he is approaching or going away from the landing field.

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MEMBERSHIP NOTES.

Changes of Address.

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Mr. E. T. Turner, Scarborough Street, Hartford, Conn.

LIBRARY NOTES.

RECENT ADDITIONS.

American Institute of Refrigeration. Proceedings Seventeenth Annual Meeting. 1928.

Antoniadi, E. M. La Planète Mars. 1930.

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MELLOR, J. W. A Comprehensive Treatise on Inorganic and Theoretical Chemistry. Volume 10. 1930.

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SCHMECKEBIER, LAURENCE F. The Aëronautics Branch, Department of Commerce: Its History Activities and Orgnization. 1930.

THORPE, FRANCIS NEWTON, Editor. Benjamin Franklin and the University of Pennsylvania. 1893.

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BOOK REVIEWS.

MATTER AND RADIATION. With Particular Reference to the Detection and Uses of the Infra-Red Rays. By John Buckingham, M.A., Late Open Exhibitioner in Natural Science of St. John's College, Cambridge; Assistant Director of Scientific Research, Admiralty. XII–144 pages, 15 illustrations, 8vo, London and New York Oxford University Press., 1930. Price $3.00.

A book written in a most engaging manner on a subject in which the author is greatly interested and hence succeeds in capturing the interest of the reader.

The first three chapters serve as an introduction to the last two which describe the detection and use of infra-red rays. The introductory matter consists of general treatises upon Electric Waves, Atoms and Molecules and the Electrical Structure of Atoms and the Production of Radiation. The various involved facts, explanations and theories are elucidated in a manner calculated to hold the attention of the reader. Apparently the means for detecting infra-red rays are devious and diverted. One may rely upon direction sensation, color changes, photography, expansion of materials, thermo-junctions, electrical resistance changes, radio-micrometers, radiometers and phosphorescent substances. Certain types of detectors have been developed and refined to such a degree that they will "Give indications of the heat of a candle over fifty miles away under suitable conditions."

The marvelous accomplishments made possible through the use of infra-red rays have been developed only in recent times, yet their performances surpass those attributed to Black Magic of the past ages. They may be used for the detection of invisible objects, secret signalling, burglar alarms and warning systems, fog penetration and "seeing through fog," distant control of torpedoes and aircraft, self-steering torpedoes, photography of invisible objects through fog, navi-

gation, and in medicine. Many of the uses listed above have been described in recent newspapers and periodicals. No doubt many additional applications have been secretly developed by the war departments of different nations and are being guarded most jealously. A few examples of what may be done through utilization of infra-red rays are, (*a*) detection of invisible vessels at ranges up to 6 miles; (*b*) on a dark night, detection of a man's head appearing over the edge of a trench parapet 50 yards away; (*c*) the presence of icebergs 6 miles away; (*d*) telephoning, sending the voice over an invisible beam of infra-red rays; (*e*) torpedoes that will automatically follow their target. A forceful illustration of the greater penetrating power of infra-red rays over light rays is given by two figures, p. 128 showing a scene taken by an ordinary photographic plate and by one specially sensitized. The author makes some interesting observations upon the relations between infra-red rays and climate. The book is furnished with Appendices I and II dealing respectively with "Notes on Electrons" and "Derivation of the Formula for the Balmer Series from the Basic Assumptions of Rutherford, Planck, and Bohr."

T. K. CLEVELAND.

THE STRUCTURE OF LINE SPECTRA. By Linus Pauling, Ph.D., Associate Professor of Theoretical Chemistry, California Institute of Technology and Samuel Goudsmit, Ph.D., Associate Professor of Theoretical Physics, University of Michigan. 258 pages, illustrations, 8vo. New York, McGraw-Hill Book Company, Inc., 1930. Price $3.50.

The explanation of the changes in energy within the atom, that produces the absorption and emission of radiant energy of various frequencies, has been the subject of numerous theoretical treatises by many investigators. The authors of this book are to be congratulated on their judicious selection of material from a much confused mass and for their coördination of this information in a form which is of value to physicists and chemists interested in the mechanics of atoms.

The scope of the book is given by the authors in the preface and is as follows:

"We have attempted to present in this book a description of the vector model of the atom and an account of the interpretation of line spectra in terms of it. We have aimed to write lucidly and intelligibly rather than encyclopedically, having in mind readers entirely unacquainted with the subject as well as those familiar with the empirical side of spectra but not with their interpretation. The book is designed primarily as a text-book for those working in the field of spectroscopy. The recent developments in theoretical physics have, however, caused spectroscopic facts and laws to become of importance for other fields in physics and chemistry, so that we are convinced that many chapters will be of use to workers in these fields also."

The book can hardly be recommended as an introductory text for a course on spectrum analysis, but for an advanced student in preparation for research in the field of atomic and molecular mechanics it will be of valuable and fruitful service.

JAMES BARNES.

A COURSE IN ENGLISH FOR ENGINEERS, Volume II. By Carl A. Naether, M.A., University of Southern California, in collaboration with George Francis Richardson, Ph.D., introduction by J. A. L. Waddell, D.E., LL.D. 488 pages, 8vo. Ginn & Company, New York. Price $3.00.

Volume 2 considers the engineer in his professional and business activities. Volume 1, reviewed a short time ago, embodied a course in English for the benefit of engineers engaged in preparing papers and reports. The work is part of a series edited by Dr. A. A. Potter intended for the instruction of the engineering profession. The present volume is practically entirely devoted to correspondence in connection with active business affairs, with a large portion devoted to the question of sales. "Big business" is now a dominant feature of this country and advertising is motive power for it. The whole subject of psychology in advertising has been developed to a very high degree within the last quarter century, and comparison of our newspapers and magazines with those of thirty or forty years ago would show the vast change that has taken place in the relation of business to the community. If we should go back to the much abused mid-Victorian period we would find still greater difference. How far this change has been a real uplift we cannot say. (There is, indeed, a "reel" uplift.) At any rate, it seems to be certain that it pays to advertise. The author of the book devotes considerable attention to the forms of personal letters as apart from form letters and gives examples in which replies to inquiries have failed to secure such attention of the inquirer as might have led to business, even though the particular product desired was not manufactured by the house that received the inquiry. Common experience would seem, however, to show that a great majority of business houses are alive to the importance of following up every clue that may lead to a sale.

In the forms of letters given, attention has been paid to the use of correct English. The previous volume was devoted to this question and statements in the introduction confirm the experience of teachers in other fields that there is a great neglect of attention to correct English in our professional institutes. Students who have received a full High School course, come to the professional schools in large numbers, careless in spelling, in grammar and even in the ordinary construction of sentences. The present work is very thorough in its treatment of the subject to which it is devoted, and will constitute a valuable number of the series to which it belongs.

HENRY LEFFMANN.

PHYSIQUE MODERNE, EXPOSE SYNTHETIQUE ET METHODIQUE DE LA PHYSIQUE D'AUJOURD'HUI ET DES TRAVAUX THEORIQUES ET EXPERIMENTAUX DES PLUS GRANDS PHYSICIENS CONTEMPORAINS, Ing. Gaetano Castelfranchi de l'Ecole Superieure des Ingenieurs de Milan. Traduction francaise par M. A. Quemper de Lanascol, Laureat de l'Institut. 660 pages, illustrations, 8vo. Paris, Librairie Scientifique Albert Blanchard, 1930. Price 70 fr.

Much of the vast quantity of literature which deals with the different phases of modern physics bristles with philosophic and mathematical deductions that may well prove a tax upon the capacity of the average scientific reader to follow. Unless he is fortunate in choosing a well balanced collection, much effort may be uselessly expended in acquiring a satisfactory outline of the subject. Professor

Castelfranchi performs a rare service to students of physical science in dealing comprehensively with modern physics in a historical, synthetic and deductive manner, within the reach of many technical readers. He wisely avoids plunging immediately into the new matter: relativity, the electron, quanta and the laws which have been derived from these hypotheses. He takes us first over the more familiar ground of atoms and molecules of physical chemistry, and then into light-radiations, the kinetic theory of gases, the brownian movement and molecular thermodynamics. The discussion of these topics which are presented analytically and deductively in the order of their evolution and with unsurpassed clarity, bring us quite into the borderland of modern physics. The reader is accordingly well prepared to proceed to the introduction and development which follows of modern views of physical processes with an adequate understanding of their relation to pre-atomistic theories.

A generally recognized characteristic of the physics of modern times is its approach toward a unification of physical phenomena in terms of purely mechanical considerations and the ideal of modifications of a single kind of elementary matter. Modern mechanics—relativity and mass—is properly the first chapter of the entrance into the new fields. Then follow the chapters on the usual topics of modern physics: "the electron and positive rays"; "X-rays and atomic number"; "Crystals"; "quanta"; "undulatory mechanics"; the statistical methods of Fermi and of Bose, and other related topics. In a treatise of such a wide scope as this, it is quite impossible to escape bearing heavily on mathematical processes, but unlike some of his contemporaries the author has obviously born in mind that many of his readers are not mathematical specialists and there appears to be little beyond the reach of a reader who is properly trained in the mathematics of the engineering courses of American universities. The physical significance of the analytic argument is kept constantly in view by unsparing explanation, and the numerous historical references by name and date to notable investigators add materially to the intrinsic value of the work and endow it with a vitality and compelling interest rarely found in a highly technical treatise.

With Professor Castelfranchi's treatise, modern physics may be properly said to come within measureable distance of having "arrived."

L. E. P.

THE FUNDAMENTALS OF HUMAN MOTIVATION. By Leonard T. Troland, S.B., A.M., Ph.D., Assistant Professor of Psychology, Harvard University. v–521 pages, cloth. New York, D. Van Nostrand Company, Inc., 1928. Price $5.00.

The author states correctly the fact that the subject of human motives has not been given the attention it deserves. In dealing with this subject he finds it necessary to introduce many other fields of interest which he believes to be directly related to this one.

We had occasion to review Dr. Troland's previous work, "Mystery of Mind." In that he brought together an impressive amount of data upon which to build his argument. "The Fundamentals of Human Motivation" is, in that respect, similar to the earlier work. It has 29 chapters, the following being the headings: The Problem of Motivation, A Review of Classical Doctrines, Modern Theories

Concerning Human Motivation, Doctrines Concerning Motivation in Animals, The General Nature of Neuromuscular Response, Determining Factors in Reflex Response, Hereditarily Determined Specific Responses, The Nature and Existence of Instincts, The Catalogue of Human Appetitional Instincts, The Characteristic Properties of the Cerebral Cortex, The Mechanism of Learning, Nociception, Beneception and Retroflex Action, Retroflex Action and Classical Problems, Secondary Retroflex Action, Fundamental Psychological Phenomena and Their Correlations, Doctrines of Psychological Hedonism, A Definite Hedonism of the Past, The Pleasures of Novelty and Their Correlations, The Nature and Basis of Emotional Experience, Desire and Purpose, Complexes and Their Complexity, The Ego, Complex and Its Operation, Sexual Motivation, The Conflict of Motives, Some Common Motivational Processes, Some Modern Interests, Personality, Character and Temperament, The Theory of Correct Conduct and Summary and Conclusion. From these headings one may realize the great scope of the work.

The author's own doctrine considers that "Consciousness and Matter are not in reality two separate systems, but are only conceived as such." For the working out of his doctrine he differentiates between the physiological and the purely psychological aspects and centers his attention upon the "activities going on in a restricted area of the cerebral cortex, a portion of the brain." Let us quote further what is really the gist of the whole book: "These activities are concerned with the 'adjustment' of response, which involves the attachment of a particular form of motor reaction to a particular object or 'stimulus' in the environment. The mechanism of such attachment involves the conduction of nervous excitation from the sense-organs through the cortex to the muscles, the kind of behavior which results being determined by the line of highest conductance among all alternative pathways." We do not have room to show how this teaching is made to fit in with the many and varied psychological situations. The treatment of the ego complex, the discussion of psychological hedonism, the pleasantness of novelty and the principle of use, the analysis of emotions, of motives in business, politics and religion and the action of narcotics, including liquor, are especially worthwhile sections. One notes in the earlier part of the book a very valuable review of the positions of many eminent psychologists. The "References and Notes," "Glossary" and "Index" are complete and especially satisfying to the student. We find ourselves agreeing with the author in the statement he makes in the Preface that the following-named questions, discussed in the book, "should be of interest to any conscious human being": (1) our inborn tendencies to action, (2) the means by which we learn, (3) the basis of "pleasure and pain" and the part which they play in learning, (4) the foundations of "happiness" in general, (5) the nature and operation of "instincts," such as that of sex, (6) the physiological meaning of the Freudian "complex," (7) the nature and foundations of emotional experience, (8) the explanation of typical modern interests: automobiles, radio, and the like, (9) suggestions towards a scientific treatment of the problems of ethics.

JOHN W. STOCKWELL.

NATIONAL ADVISORY COMMITTEE FOR AËRONAUTICS.

Report No. 350. Working Charts for the Selection of Aluminum Alloy Propellers of a Standard Form to Operate with Various Aircraft Engines and Bodies. By Fred E. Weick. 16 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price ten cents.

The investigation reported was conducted by the National Advisory Committee for Aëronautics at the request of the Bureau of Aëronautics, Navy Department. This is the third in a series of investigations of the water pressures on seaplane floats and hulls, and completes the present program. It consisted of determining the water pressures and accelerations on a Curtiss H-16 flying boat during landing and taxying maneuvers in smooth and rough water.

The results show that the greatest water pressures occur near the keel at the main step, where the maximum pressure is approximately 15 pounds per square inch. From this point maximum pressures decrease in magnitude toward the bow and chine. Pressures of approximately 11 pounds per square inch were experienced at the keel slightly forward of the middle of the forebody when taking off in rough water. The area of the forebody subjected to considerable pressure is roughly a triangle having its base at the step and its apex on the keel at the load water line forward. On the bottom between steps, a maximum pressure of 8 pounds per square inch is nearly uniform. A vertical acceleration of 4.7 g is the greatest value encountered in landings, and is considerably greater than any other value recorded. It was found that 3 g is approximately the maximum to be expected in take-offs in rough water, and that this value was exceeded during only a few landings. A longitudinal acceleration of 0.9 g was once attained in a landing in rough water and 0.7 g is not unusual for take-offs in rough water. The maximum lateral acceleration attained in cross-wind landings is approximately 0.5 g. the results show that the landing loads were usually borne by an area near the main step, and that rough water may cause large loads to be applied near the middle of the forebody.

Report No. 346. Water Pressure Distribution on a Flying Boat Hull. By F. L. Thompson. 18 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price ten cents.

Working charts are given for the convenient selection of aluminum alloy propellers of a standard form, to operate in connection with six different engine-fuselage combinations. The charts have been prepared from full-scale test data obtained in the 20-foot propeller research tunnel of the National Advisory Committee for Aëronautics. An example is also given showing the use of the charts.

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Strength of Materials, by S. Timoshenko. Part I, Elementary theory and problems; Part II, advanced theory and problems. 2 volumes, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1930. Price $8.00.

General Physics, by Wm. S. Franklin, Sc.D., and G.E. Grantham, Ph.D. 705 pages, illustrations, 8vo. Lancaster, Pa., Franklin and Charles, 1930. Price $4.00.

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National Advisory Committee for Aëronautics. Technical Notes No. 346. Span Load Distribution on Two Monoplane Wing Models as Affected by Twist and Sweepback, by Montgomery Knight and Richard W. Noyes. 7 pages, plate tables and diagrams, quarto. Washington, Committee, 1930.

Bell Telephone Laboratories, Monographs: No. B-488, Preparation of Air of Known Humidity and Recorder Calibration, by A. C. Walker and E. J. Ernst, Jr., 12 pages, illustrations, 8vo. B-489, Moisture Content of Compressed Nitrogen, by A. C. Walker and E. J. Ernst, Jr., 4 pages, illustrations, 8vo. B-490, Developments in Communication Materials, William Fondiller, 21 pages, illustrations, 8vo. B-491, The Words and Sounds of Telephone Conversations, by N. R. French, C. W. Carter, Jr., and Walter Koenig, Jr., 35 pages, illustrations, 8vo. B-492, The Reciprocal Energy Theorem, by John R. Carson, 7 pages, 8vo. B-493, The Approximate Networks of Acoustic Filters, by W. P. Mason, 9 pages illustrations, 8vo. B-494, Some Contemporary Advances in Physics—XX Ionization of Gases by Light, by Karl K. Darrow, 15 pages, illustrations, 8vo. B-495, Motion of Telephone Wires in Wind, by D. A. Quarles, 8 pages, illustrations, 8vo. B-496, Economic Quality Control of Manufactured Product, by W. A. Shewhart, 26 pages, illustrations, 8vo. B-498, The Chromolinoscope Revived, by Herbert E. Ives, 11 pages, illustrations, 8vo.

Les Quanta, par Georges Dejardin, Collection Armand Colin. 224 pages, illustrations, 16mo. Paris, Librairie Armand Colin, 1930. Price, in paper, 10 fr. 50.

United States Coast and Geodetic Survey, Special Publication No. 164, First-Order Triangulation in Southeast Alaska, by Walter F. Reynolds. 157 pages, illustrations, maps. Washington, Government Printing Office, 1930. Price 40 cents.

Les Ondes Electriques de Tres Courtes Londueurs et Leurs Applications, by C. Gutton. 20 pages, illustrations, 8vo. Paris, Librairie Scientifique Hermann et Cie, 1930. Price 4 frs.

L'Electro-Acoustique Rapports de L'Acoustique Moderne et de L'Electricite, par Pierre David. 39 pages, illustrations, 8vo. Paris, Librairie Scientifique Hermann et Cie, 1930. Price 5 frs.

Cours D'Analyse Professe a L'ecole Polytechnique, par J. Hadamard. Tome Second, 721 pages, illustrations, 8vo. Paris, Librairie Scientifique Hermann et Cie, 1930. Price, in paper, 140 frs.

Grundbegriffe der Chemie, von Dr. E. Rabinowitsch. 151 pages, 16mo. Berlin, Walter de Gruyter & Company, 1930. Price 1 M. 80.

A Course in English for Engineers, Volume II, by Carl A. Naether, M.A., University of Southern California, in collaboration with George Francis Richardson, Ph.D., introduction by J. A. L. Waddell, D.E., LL.D. 488 pages, 8vo. Ginn & Company, New York.

Canada, Department of Mines, Mines Branch. Investigations of Mineral Resources and the Mining Industry, 1928. 57 pages, pl. maps, 8vo. Ottawa, King's Printer, 1930.

The Fundamental Theory of Electrical Engineering, by Arthur Lemuel Albert,

M.S., 323 pages, illustrations, tables, 8vo. Boston, Ginn and Company, 1930. Price $3.20.

United States Department of Commerce, Bureau of Mines. Mineral Resources of the United States, 1927, Part I—Metals, Part II—Nonmetals, 2 volumes, 8vo. Washington, Government Printing Office, 1930.

Laboratory Manual of Physics, by Clinton Maury Kilby, Ph.D. 129 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1930. Price $1.75.

CURRENT TOPICS.

The Discovery of the Law of Conservation of Energy. (*Isis*, No. 40.) *Isis* is from time to time publishing not merely the text of certain scientific classics but photographic facsimiles of them, saying that "the greatest bane of our studies is their secondhandedness." de Moivre's work on the binomial theorem, Pascal on conics, and two papers by Oersted in Latin and English respectively have already appeared. Now Robert Mayer's "Remarks on the Forces of Inorganic Nature" is presented.

At almost the same time the discovery of a fixed numerical relation between heat and work was made by two very different men, Julius Robert Mayer (1814–1878) and James Prescott Joule (1818–1889). Mayer, a German physician on a Dutch ship, discovered the law of conservation of energy by a sudden intuition. This great discovery, comparable in its suddenness to a religious conversion, occurred while he was in the harbor of Surabaya (N. E. Java) in July, 1840. His whole subsequent life was devoted to the furtherance of his great conception. Under the strain his health gave way and he was confined in an asylum for the insane.

His first paper on the subject, entitled "On quantitative and qualitative determination of forces," was written in June, 1841. Poggendorff rejected it, a fortunate circumstance because it had serious errors. Another paper composed half a year later, the "Remarks" reproduced in this number of *Isis*, was accepted by Liebig and published in the *Annalen der Chemie und Pharmacie* in 1842. "This was the first paper containing a clear statement of the law of conservation of energy, and an attempt to determine the mechanical equivalent of heat." An English translation of it was printed in the *Philosophical Magazine* in 1862 at the desire of Tyndall and Joule. Mayer developed his views in many subsequent papers. Three of his important discussions were published, not in journals, but independently by the author. There are three editions of his collected works, one of which contains the rejected paper of 1841.

In Mayer's *Remarks* the word "force" is often used whereas "energy" does not appear, scientific usage not yet having adopted it. Mayer closely scans the causal connection between the ceasing of motion and the appearance of heat. He examines the possibility of the heat developed by friction being due to a lessening of the volume of the bodies in contact. "The heat cannot be derived from

the diminution of the volume of the rubbing substances. Water undergoes, as found by the author, a rise of temperature when violently shaken. The water so heated (from 12° to 13° C.) has a greater bulk after being shaken than it had before; whence now comes this quantity of heat, which by repeated shaking may be called into existence in the same apparatus as often as we please?"

Mayer reaches a definite conclusion. "If falling force and motion are equivalent to heat, heat must also naturally be equivalent to motion and falling force." He poses the experimental problem very clearly thus: "How great is the quantity of heat which corresponds to a given quantity of motion or falling force? For instance, we must ascertain how high a given weight requires to be raised above the ground in order that its falling force may be equivalent to the raising of the temperature of an equal weight of water from 0° to 1° C. The attempt to show that such an equation is the expression of a physical truth may be regarded as the substance of the foregoing remarks." He does not, however, stop with the formulation of the experimental problem. He proceeds to calculate the mechanical equivalent of heat by using the ratio between the specific heats of air at constant pressure and at constant volume. His result is that "the warming of a given weight of water from 0° to 1° C. corresponds to the fall of an equal weight from the height of about 365 meters. If we compare with this result the working of our best steam-engines, we see how small a part only of the heat applied under the boiler is really transformed into motion or the raising of weights; and this may serve as justification for the attempts at the profitable production of motion by some other method than the expenditure of the chemical difference between carbon and oxygen—more particularly by the transformation into motion of electricity obtained by chemical means."

Mayer's intuitional discovery was followed by the manifold and exact measurements of Joule in England. His first paper on the subject was read before the chemical section of the B. A. A. A. at Cork in 1843 and was printed in the *Philosophical Magazine* in the same year. Mayer's first paper appeared in a chemical journal and Joule presented his results first to the chemical section. "We may thus say that the greatest generalization of nineteenth century physics was introduced to the scientific world through the chemical door." Joule fared better at the hands of his contemporaries than did Mayer. In 1850 he entered the Royal Society, while Mayer received no recognition till 1858 and then by election to the less important Scientific Society of Basle. Unfortunately a controversy arose with regard to the relative merits of Mayer and Joule. On the side of Mayer were Schönbein, Liebig, Dühring and John Tyndall

while arrayed against him were P. G. Tait. Lord Kelvin and Helmholtz. G. Sarton, the editor of *Isis* holds that the former group was fairer to Joule than the latter were to Mayer. "These two names, Mayer and Joule," says *Isis*, "must remain united in our grateful hearts. Mayer's views were vague but broad and he was the first. Joule reached the same general conclusion a little later but after having followed an entirely different path. The one thing that might be said to Joule's advantage is this: Mayer's work was in itself insufficient, it could not stand without the experimental confirmation which Joule provided; on the contrary, Joule's achievement was independent of Mayer's and needed no external support."

Both Mayer and Joule were anticipated by Sadi Carnot (1796–1832), who discovered the second Law of Thermodynamics. His manuscripts becoming accessible in 1878 showed that he had the idea of the First Law with remarkable clearness and had deduced the relation that one calorie equals 370 kilogrammeters. As he did not publish his results he does not receive credit, according to the custom of the scientific world. Séguin (1786–1875) in 1839 in print showed that he had some conception of the proportionality of heat and work but he made no endeavor to determine the exact relation between them. The Dane, L. A. Colding, likewise in 1843 presented to the Academy of Sciences of his country a memoir in which he expressed views resembling those of Mayer, and recounted some experiments as well, but his paper was not published till 1856.

G. F. S.

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